Lecture 2

- This lecture, we will look at:
  - Electric fields
  - Electric field lines
  - The electric field due to a dipole
  - How a dipole behaves in an electric field

- After this lecture, you should be able to answer the following questions:
- A 0.03 C point charge is placed at the origin of a coordinate system. What is the direction and magnitude of the electric field due to this charge:
  - At the point (3.0 m, 4.0 m)?
  - At the point (-3.0 m, -4.0 m)?
  - At the point (6.0 m, 8.0 m)?

Electric Fields

- How does one charge affect a second charge – “action at a distance”?
- Consider that one charge produces an electric field which then influences the second charge.
- Electric field strength is the force per unit charge:

  $\vec{E} = \vec{F}/q_0$ [2.1]
- Note, the electric field is a vector field which determines the magnitude and direction of the force acting on (test) charges within it:

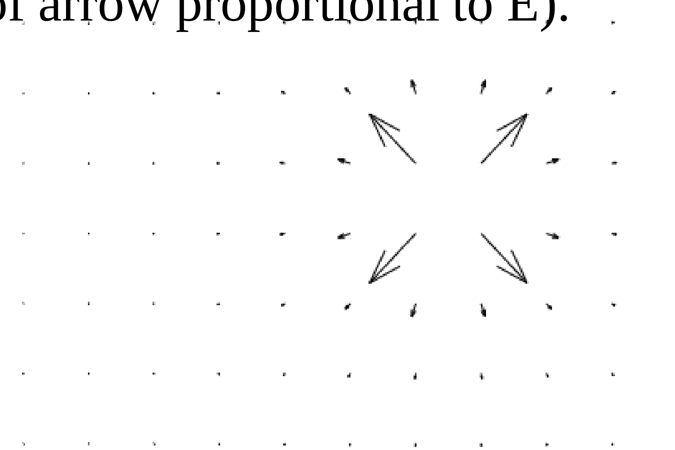
  $\vec{F} = q\vec{E}$ [2.2]
- Units of electric field, $N\,C^{-1}$.

- Electric field due to point charge q:

$$E = \frac{F}{q_0} = \frac{1}{q_0}\frac{1}{4\pi\varepsilon_0}\frac{q\,q_0}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad [2.3]$$

- Map of field for point charge (length of arrow proportional to E).



- What is sign of charge in diagram?

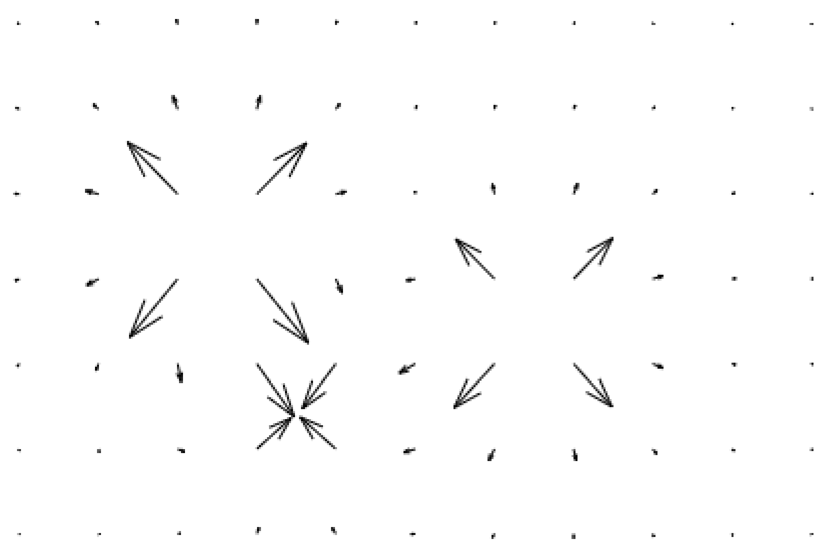
Principle of Superposition

- Force on q_0 due to charges $q_1 ... q_n$:

  $\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + ... + \vec{F}_{0n}$.
- Hence E field due to charges 1...n is:

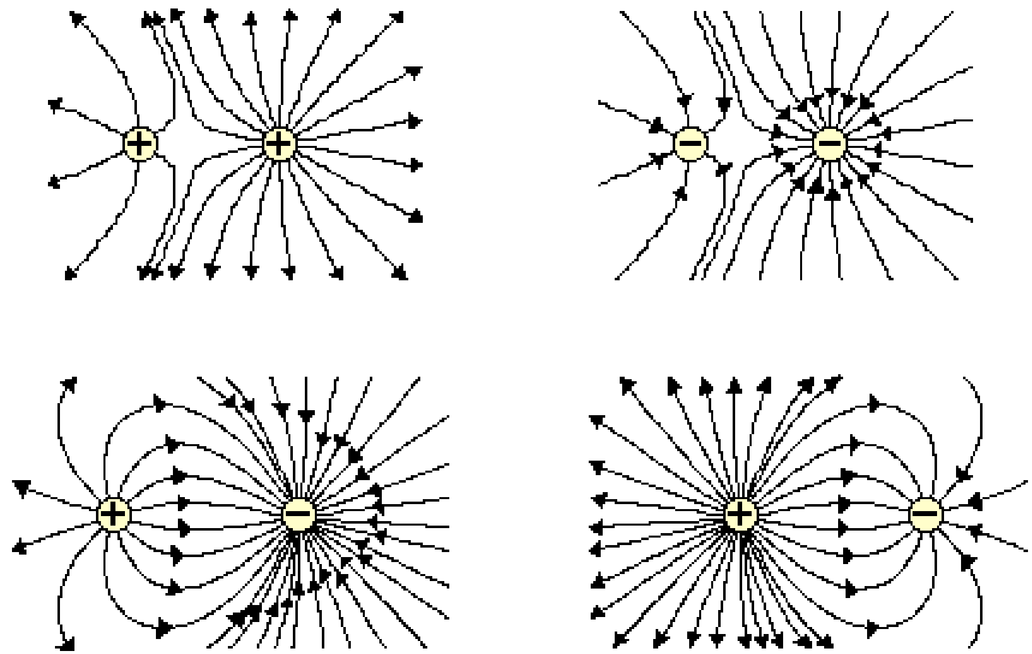
$$\vec{E} = \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + ... + \frac{\vec{F}_{0n}}{q_0}$$

$$= \vec{E}_1 + \vec{E}_2 + ... + \vec{E}_n \qquad [2.4]$$



Electric Field Lines

- Electric field lines are way of visualising E fields.
- Lines trace path followed by (slow) test charge, density of lines proportional to field strength.
- Examples, charges not balanced!



Electric Field due to Dipole

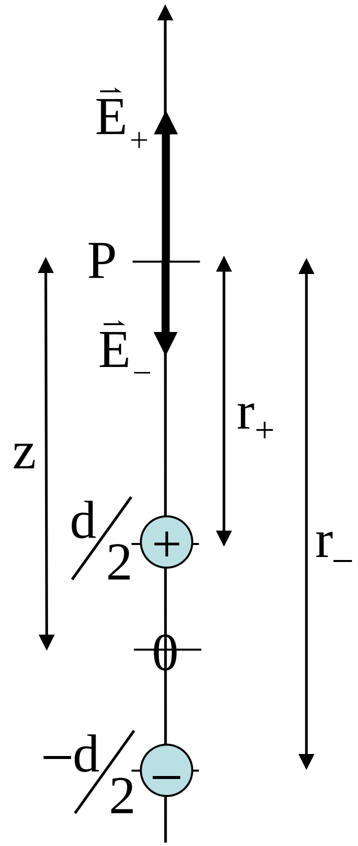
- Dipole consists of two equal but opposite charges $\pm q$ separated by a distance d.
- Determine field at point P on dipole axis (z axis):

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$$E = E_+ - E_-$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+^{\,2}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_-^{\,2}}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(z-\frac{d}{2})^2} - \frac{1}{(z+\frac{d}{2})^2}\right).$$



- Rearrange so can use binomial theorem:

$$E = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{z^2(1-\frac{d}{2z})^2} - \frac{1}{z^2(1+\frac{d}{2z})^2}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{1}{z^2}\left((1-\tfrac{d}{2z})^{-2} - (1+\tfrac{d}{2z})^{-2}\right).$$

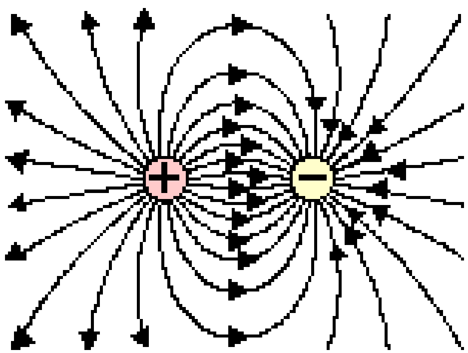
- For z>>d:

$$E = \frac{q}{4\pi\varepsilon_0}\frac{1}{z^2}\left((1+\tfrac{d}{z}) - (1-\tfrac{d}{z})\right)$$

$$= \frac{q}{2\pi\varepsilon_0}\frac{d}{z^3}.$$

Electric Field due to Dipole

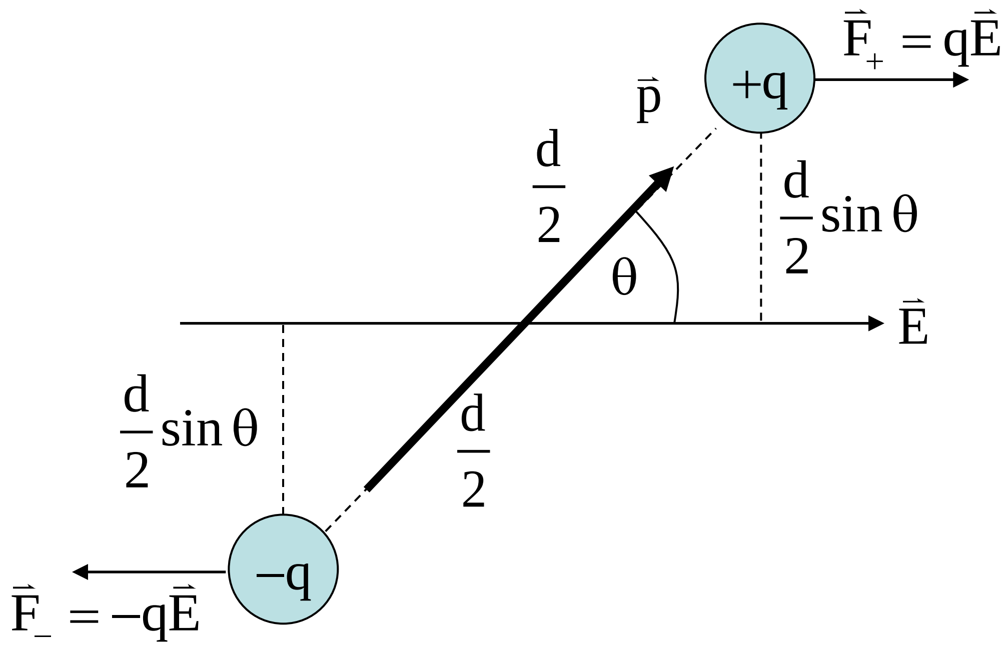
- At large distance, field due to dipole depends on product q x d.



- Define vector quantity, the electric dipole moment, $\vec{p}$.
- Magnitude p = q d [2.5]
- Direction from –ive to +ive.
- Units of dipole moment C m.
- For point charge $E \propto 1/r^2$.
- For dipole $E \propto 1/r^3$.

Torque on Dipole in an Electric Field

- Consider dipole in uniform E field:



- No net translational force.
- Torque: $\tau = qE\frac{d}{2}\sin\theta + qE\frac{d}{2}\sin\theta$

$$= pE\sin\theta \qquad [2.6]$$