The Standard Model

1: Overview / introduction

Course Outline

Lectures:

1. Overview of SM framework (2 lectures)
2. Tests of SM with Z^0 bosons
3. Test of SM with $W^{\pm}$ bosons
4. Quark mixing and CP violation (2 lectures)

Outline

Overview of Standard Model:

- What does it describe?
- What does it need as inputs?
  - What do we use and why
- What can it predict?
- Where can it be tested?

Overview

Standard model describes interactions of fundamental particles:

- **Fermions**; 6 flavours of quarks (3 types - rgb), leptons
- **Vector bosons**; γ, $W^{+/-}$, Z^0, 8 gluons
- **Scalar bosons**; H

Theory describes at least 61 fundamental particles and three forces!

Overview

- SM unites electromagnetic, strong and weak forces
- Represented theoretically by
  - U(1) hypercharge
  - SU(3) colour
  - SU(2) isospin
- Use lagrangian to describe particle field and interactions

Lagrangians

- Describe interactions and fields
- Classically, L = kinetic energy – potential energy
- Particle physics:
  - Use **Dirac equation** to describe **free** spin-1/2 particle:

$$L = \bar{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi$$

Ψ = wavefunction
m = mass
γ^{μ} = μ^{th} gamma matrix
∂_{μ} = partial derivative

Symmetries

Note: symmetries in physics imply conservation laws

symmetry	**invariant**
Space translation	momentum
Time translation	energy
Rotation	Angular momentum
Global phase; $\Psi \to e^{i\theta}\Psi$	Electric charge
Local phase; $\Psi \to e^{i\theta(x,t)}\Psi$	Lagrangian + gauge field ($\to$ QED)

Lagrangians

Apply local gauge symmetry to Dirac equation:

$\bar{\Psi} \to e^{i\theta(x,t)}\bar{\Psi}$, $\Psi \to e^{-i\theta(x,t)}\Psi$

Consider very small changes in field:

$\Psi \to \Psi + \delta\Psi = \Psi - i\theta(x,t)\Psi$ ie. $\delta\Psi = -i\theta(x,t)\Psi$

$L = \bar{\Psi}(i\gamma^\mu\partial_\mu - m)\Psi \Rightarrow \delta L = \bar{\Psi}\gamma^\mu\partial_\mu\theta(x,t)\Psi$

If lagrangian is invariant then $\delta L=0$

Lagrangians

Invariant lagrangian $\Rightarrow \delta L=0$

Satisfied:

1) if we introduce gauge field A_μ to interact with fermion, and A_μ transforms as;

$$A_\mu + \delta A_\mu = A_\mu + 1/e\,\partial_\mu \theta(x,t)$$

2) If we replace $\partial_\mu \rightarrow D_\mu = \partial_\mu + ieA_\mu$

Hence $L = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi$ is invariant

You can check this ....

Lagrangians

Not the whole story – need to add term for field strength (kinetic term):

Define $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
Add term $-1/4 F_{\mu\nu}F^{\mu\nu}$ (Lorentz invariant, matches Maxwell's equations)

Final lagrangian (for QED!):

$$L = -1/4 F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

Nb. No mass term for A_μ allowed; then L is not invariant

QED from gauge invariance requirement + massless γ!

QED

Electromagnetic interactions; abelian

- U(1): 1 gauge field $\mathbf{B}_\mu$, coupling g'
  - In SM, source is hypercharge Y
  - Field strength $\alpha_{em} = g'^2/4\pi$ (fine structure constant)
    $\alpha_{em} = 1/137$

$$\mathbf{L_Y = -1/4F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi}$$

where $\mathbf{D_\mu} = \partial_\mu + ieB_\mu$
to conserve invariance under $\Psi \rightarrow e^{-iY(\Psi)}\Psi$

Strong, weak lagrangians

Strong, weak forces are described by **non-abelian theories:**

- In non-abelian theories gauge bosons can self interact
- In non-abelian theories gauge invariance achieved by adding n^2-1 gauge bosons for SU(n)
  - SU(2) – 3 gauge bosons (W^1, W^2, W^3) for weak force
  - SU(3) – 8 gluons for QCD

QCD

Strong interactions; non-abelian

- SU(3): 8 massless gauge bosons (gluons), coupling g_s
  - Source is colour
  - Field strength $\alpha_s = g_s^2/4\pi$
- Fields represented by quark triplets (3 colours)
  - Hadrons (observable states) colourless
  - Leptons, neutrinos do not couple to gluons

QCD

$$\mathbf{L_{QCD} = -1/4F^{(a)}_{\mu\nu}F^{(a)\mu\nu} + \bar{\Psi}^j (i\gamma^\mu(D_\mu)_{jk} - m_{jk})\Psi_k}$$

where:

$$\mathbf{D_\mu = \delta_{jk}\partial_\mu + ig_s\Sigma_a (\lambda^a_{j,k}/2)G^a_\mu}$$

λ^a are 8 3x3 matrices (analogous to Pauli spin matrices in SU(2))

a is sum over 8 gauge bosons (section 35, PDG)

j,k is colour index (=1,2,3)

G^a_μ is (a^th) gluon gauge field

$F_{\mu\nu}F^{\mu\nu}$ contains triple, quartic couplings

$$\mathbf{F^{(a)}_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f_{abc} G^b_\mu G^c_\nu}$$

f_{abc} structure constants for SU(3) (section 35, PDG)

Weak force

Weak interactions: non-abelian

- SU(2): 3 gauge bosons W^1, W^2, W^3
  - coupling strength g
  - Source is weak charge
- **Observed** to violate parity so left, right handed fermions treated separately in theory

$\Psi = \Psi_L + \Psi_R$

Ψ_L **doublet**; interacts under weak force

Ψ_R **singlet**, does not interact under weak force

nb: $\Psi_{L/R} = \frac{1}{2}(1 \mp \gamma^5)\,\Psi$ in theory

Weak force

$$L_{WEAK} = -1/4 F^{(a)}_{\mu\nu} F^{(a)\mu\nu} + i\bar{\Psi}_L(\gamma^\mu(D_\mu) - m)\Psi_L$$

where:

$D_\mu = \partial_\mu + igT^a W^a_\mu$

a is sum over 3 gauge bosons

T^a = 0.5*Pauli spin matrices (PDG!)

W^a_μ is weak gauge boson field

$$F^{(a)}_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g_s \varepsilon_{abc} W^b_\mu W^c_\nu$$

ε_{abc} = SU(2) structure constants

$F_{\mu\nu}F^{\mu\nu}$ contains triple, quartic couplings

Combine forces in SM

Left handed fermions:

Interact with weak force
Interact with electromagnetic force
Quarks interact with strong force

Right handed fermions:

Interact with electromagnetic force
Quarks interact with strong force

nb: strong quark eigenstates are superposition of weak quark eigenstates: → CKM matrix (later)

Full SM lagrangian

SM: U(1) x SU(2) x SU(3)

Substitute in for D_μ, $F_{\mu\nu}$ for each interaction, and fermion field $\Psi = \Psi_L + \Psi_R$

Free particle term

$$\mathcal{L}_{(SM,1)} = \sum_{\text{gauge bosons}} -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{\text{fermions}} \bar{\psi}\, \not{D} \psi + (D_\mu \Phi)^{+} (D^\mu \Phi) - V(\Phi)$$

Gauge boson interaction terms

Higgs field Φ terms

(to give mass)

$$V(\Phi) = -\mu^2 \Phi^{+}\Phi + \lambda \left|\Phi^{+}\Phi\right|^2$$

Note; Masses

- Everything in SM lagrangian is **massless**
- Mass conferred via **Higgs mechanism**
  1. Introduce Higgs field (doublet) $\Phi = (0, v+H)$
  2. Substitute into lagrangian
  3. $D_\mu\Phi D^\mu\Phi$ contains terms of the form

     $\mathbf{(g^2v^2/4)W_\mu^+W_\mu^- + (v^2/8)(gW_\mu^3 - g'B_\mu)^2}$

     where $\mathbf{W_\mu^+} = (W_\mu^1 + iW_\mu^2)/\sqrt{2}$ etc.

     $\mathbf{v}$ = vacuum expectation value (246 GeV)

  $\Rightarrow$ Masses for gauge bosons

  Mixing of U(1) and SU(2) gauge bosons (electroweak unification)

→ Ronan

Electroweak unification

W_μ^1, W_μ^2 mix after Higgs breaking → massive W^+ and W^-

$W^+ = (W_\mu^1 + W_\mu^2)/\sqrt{2}$ (massive)

$W^- = (W_\mu^1 - W_\mu^2)/\sqrt{2}$ (massive)

W_μ^3, B_μ mix after Higgs breaking → massive Z^0, γ

$Z^0 = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu$ (massive)

$\gamma = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3$ (massless)

Unification here!

SM relates M_W, M_Z and g',g:

$\tan\theta_W = g' / g$

$M_W = M_Z{*}\cos\theta_W$

Unification here!

Effect on W±, γ couplings

Subst for W^+,W^-,Z^0,γ in lagrangian;

- Couplings for $\mathbf{W^\pm}$ of form (only l.h. fermions)

  $-g/2\sqrt{2}\, \bar{\nu}\gamma_\mu(1-\gamma^5)eW_\mu^-$

- Couplings for **photons** of form (l.h. + r.h. charged fermions)

  $g \sin\theta_W \bar{e}\gamma_\mu e B_\mu$

γ_μ transforms as vector (P odd)
$\gamma_\mu\gamma^5$ transforms as axial vector (P even)
(Vector + axial vector ⇒parity violation for weak force)

You can check this ….

Effect on couplings

Subst for W^+,W^-,Z^0,γ in lagrangian;

- Couplings for $\mathbf{Z^0}$ of form (l.h. (W_μ^3) and r.h. (B_μ) fermions)

  $-g/4\cos\theta_W\, \bar{\nu}\gamma_\mu(1-\gamma^5)\nu Z_\mu$ (lh neutrinos)

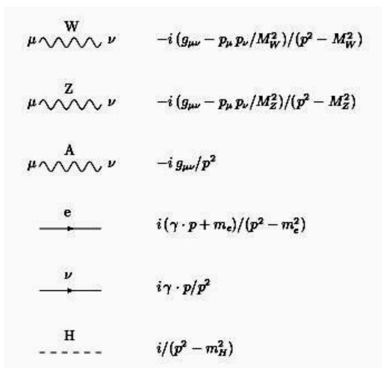
  $g/4\cos\theta_W\, \bar{e}(\gamma_\mu(1-\gamma^5)-4\sin^2\theta_W\gamma_\mu)eZ_\mu$ (lh & rh e)

⇒ Effective vector and axial couplings g_{vf}, g_{af} for Z^0 decays; $g/4\cos\theta_W\, e(\gamma_\mu(g_{vf} - g_{af}\gamma^5))eZ_\mu$

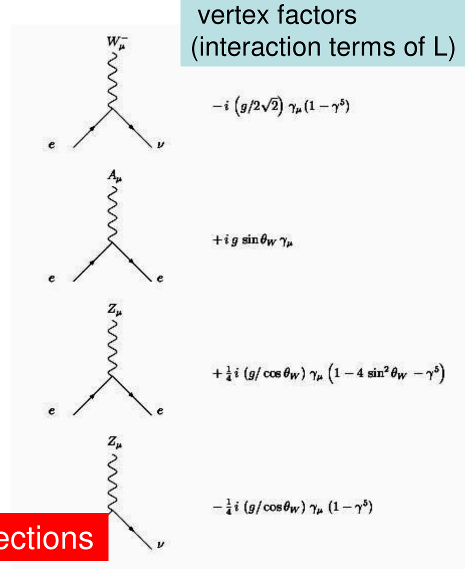
fermion	g_{vf}	g_{af}
$\nu e,\nu\mu,\nu\tau$	0.5	0.5
e,μ,τ	$-0.5+2\sin\theta_W$	-0.5
u,c,t	$0.5-4/3\sin^2\theta_W$	0.5
d,s,b	$-0.5+2/3\sin^2\theta_W$	-0.5

Feynman rules

Propagators (free particle L)



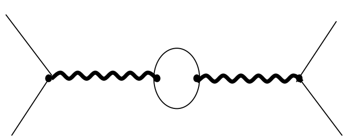
vertex factors (interaction terms of L)



Allow us to calculate cross-sections

Running coupling constants

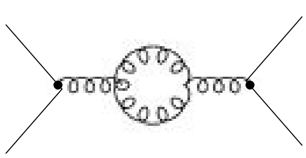
- Arise from loop diagrams "screening" charge at large distance scales:
  - $\Rightarrow$ Coupling constant values are function of E



Screening of charge by vacuum polarisation;

High E $\Rightarrow$smaller distance scale $\Rightarrow$see more charge

Coupling constant increases with E



Non-abelian forces also include these "extra" charge loops

Net effect: coupling constant decreases with E (asymptotic freedom)

How many free parameters?

SM does not predict:

- Magnitude of gauge couplings g, g', g_s
- Masses of fermions (3 leptons, 6 quarks, 3 neutrinos)
- Weak-strong eigenstate quark mixing (can express by 4 parameters), ditto for neutrinos
- Higgs related quantities (mass of Higgs and vacuum expectation value)

$\Rightarrow$ **Some 26 unknowns in the theory.**

- These values must be added by hand (experimental measurements)

SM predicts relationships

All observables can be predicted in terms of 26 free parameters

- If we have > 26 measurements of these observables, we overconstrain SM
- Overconstrain $\Rightarrow$ we don't have any more ad hoc inputs AND we can test the consistency of the model

- **Best plan:**
  - pick well measured set of observables
  - Calculate other observables in terms of these well known quantities
  - Test predictions; measure observable, compare to theory

SM predicts relationships

Due to predicted relationships, can reduce number of unknowns

- For example, electroweak boson masses and couplings **M_w,M_z,M_γ,g,g':**
  - $\mathbf{g}.\sin\theta_w = \mathbf{g'}.\cos\theta_w = e$
  - $\mathbf{M_\gamma} = 0$
  - $\mathbf{Mw} = \mathbf{Mz}.\cos\theta_w$

→ 3 parameters free (g, $\sin\theta_w$, Mw)

- In practise, pick 3 that are best measured experimentally:
  α_{em}, G_F, Mz

Where can the SM be tested?

Particle physics experiments designed to test specific aspects of SM

- Major historical experiments:
  - LEP (ALEPH,DELPHI,L3,OPAL) **Electroweak, qcd**
    ($\sqrt{s} = M_Z \rightarrow > 2M_W$)
- Major running experiments:
  - Babar, Belle ($\sqrt{s} = 2M_B$) **Quark mixing, CP**
  - CDF, D0 ($\sqrt{s}$ = 2 TeV) **electroweak, qcd, quark mixing**
  - H1, Zeus ($\sqrt{s}$ = 0.3 TeV) **qcd**

Review

SM unites electromagnetic, weak, strong forces

SM predicts cross-sections, couplings

SM incomplete – 26 free parameters

- Relations between some free parameters are predicted

Next lectures will discuss how these free parameters are measured and SM is tested