The Standard Model

Overview / introduction

Outline

- Overview of Standard Model:
  - What does it describe?
  - What does it need as inputs?
    - What do we use and why
  - What can it predict?
  - How is it tested and how well does it perform?

Overview

Standard model describes interactions of fundamental particles:

- **Fermions**; 6 flavours of quarks (3 types - rgb), leptons
- **Vector bosons**; γ, $W^{+/-}$, Z^0, 8 gluons
- **Scalar bosons**; H

Theory describes at least 61 fundamental particles and three forces!

(note: not gravity – SM is incomplete at the outset)

Overview

- SM unites electromagnetic, strong and weak forces
- Represented theoretically by
  - U(1) hypercharge
  - SU(3) colour
  - SU(2) isospin
- Use lagrangian to describe particle field and interactions

Lagrangians

- Describe interactions and fields
- Classically, L = kinetic energy – potential energy
- Particle physics:
  - Use Dirac equation to describe free spin-1/2 particle:

$$L = \bar{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi$$

Symmetries

Note: symmetries in physics imply conservation laws

symmetry	**invariance**
Space translation	momentum
Time translation	energy
Rotation	Angular momentum
Global phase; $\Psi \rightarrow e^{i\theta}\Psi$	Electric charge
Local phase; $\Psi \rightarrow e^{i\theta(x,t)}\Psi$	SM lagrangian

Lagrangians

Apply local gauge symmetry to Dirac equation:

$\bar{\Psi} \rightarrow e^{i\theta(x,t)}\bar{\Psi}$, $\Psi \rightarrow e^{-i\theta(x,t)}\Psi$

Consider very small changes in field

$\Psi \rightarrow \Psi + \delta\Psi = \Psi - i\theta(x,t)\Psi$ ie. $\delta\Psi = i\theta(x,t)\Psi$

$L = \bar{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi \Rightarrow \delta L = \bar{\Psi}\gamma^{\mu}\partial_{\mu}\theta(x,t)\Psi$

Invariant lagrangian $\Rightarrow \delta L=0$

Satisfied 1) if we introduce gauge field A_μ to interact with fermion, and A_μ transforms as;

$$A_\mu + \delta A_\mu = A_\mu + 1/e\,\partial_\mu\theta(x,t)$$

2) If we replace $\partial_\mu \rightarrow D_\mu = \partial_\mu + ieA_\mu$

Lagrangians

Hence $L = \overline{\Psi}(i\gamma^{\mu}D_{\mu} - m)\Psi$ **is** invariant

Not the whole story – need to add term for field strength (kinetic term):

Define $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$

Add term $-1/4F_{\mu\nu}F^{\mu\nu}$ (Lorentz invariant, matches Maxwell's equations)

Final lagrangian (for QED!):

$$L = -1/4F_{\mu\nu}F^{\mu\nu} + \overline{\Psi}(i\gamma^{\mu}D_{\mu} - m)\Psi$$

Nb. No mass term for A_{μ}; then L is not invariant

QED

Electromagnetic interactions; abelian

- U(1): 1 gauge field B_μ, coupling **g'**
  - In SM, source is hypercharge Y
  - Field strength $\alpha_{em} = g'^2/4\pi$ (fine structure constant) = 1/137

$$L_Y = -1/4 F_{\mu\nu}F^{\mu\nu} + \overline{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

$$D_\mu = \partial_\mu + ieB_\mu$$

to conserve invariance under $\Psi \rightarrow e^{-iY(\Psi)}\Psi$

(value not predicted)

Lagrangians

- Strong, weak forces are described by **non-abelian theories:**
    - In non-abelian theories gauge bosons can self interact
    - In non-abelian theories gauge invariance achieved by adding n^2-1 gauge bosons for SU(n)

QCD

Strong interactions; non-abelian

- SU(3): 8 massless gauge bosons (gluons), coupling **g_s**
  - Source is colour
  - Field strength $\alpha_s = g_s^2/4\pi$
- Fields represented by quark triplets (3 colours)
  - Hadrons (observable states) colourless
  - Leptons, neutrinos do not couple to gluons

QCD

$$L_{QCD} = -1/4F^{(a)}{}_{\mu\nu}F^{(a)\mu\nu} + i\Sigma_q \bar{\Psi}^i{}_q(i\gamma^\mu(D_\mu)_{ij} - m)\Psi_{jq}$$

$$D_\mu = \delta_{ij}\partial_\mu + ig_s\Sigma_a (\lambda^a{}_{i,j}/2)A^a{}_\mu$$

λ^a are 8 3x3 matrices (analogous to Pauli spin matrices in SU(2))

a is sum over gauge bosons (section 35, PDG)

$A^a{}_\mu$ is gluon gauge field

$$F^{(a)}{}_{\mu\nu} = \partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu - g_s f_{abc} A^b{}_\mu A^c{}_\nu$$

f_{abc} structure constants for SU(3) (section 35, PDG)

Weak force

Weak interactions: non-abelian

- SU(2): 3 gauge bosons W_μ^1, W_μ^2, W_μ^3, coupling **g**
  - Source is weak charge
- **Observed** to violate parity so left, right handed fermions treated separately in theory

$\Psi = \Psi_L + \Psi_R$

Ψ_L **doublet**; interacts under weak force

$D_\mu = \partial_\mu + igT^a W^a_\mu$

T^a = Pauli spin matrices

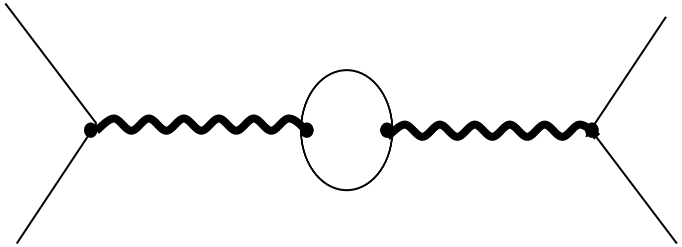
Ψ_R **singlet**, does not couple to $W_\mu^1, W_\mu^2, W_\mu^3$

note: implies zero fermion mass ($\Psi m \Psi$ independent of handedness, and would give contribution from right-handed state)

Aside: Running coupling constants

- Arise from loop diagrams "screening" charge at large distance scales:

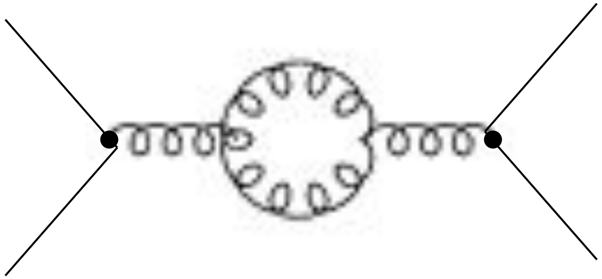
  ⇒ Coupling constant values are function of E



Screening of charge by vacuum polarisation;

High E ⇒smaller distance scale ⇒see more charge

Coupling constant increases with E



Non-abelian forces also include these "extra" charge loops

Net effect: coupling constant decreases with E

Combine forces in SM

Left handed fermions:

Interact with weak force
Interact with electromagnetic force
Quarks interact with strong force

Right handed fermions:

Interact with electromagnetic force
Quarks interact with strong force

Note: strong quark eigenstates are superposition of weak quark eigenstates: → CKM matrix (later)

SM lagrangian

SM: U(1) x SU(2) x SU(3)

Substitute in for D_μ, $F_{\mu\nu}$ for each interaction, and fermion fields

Free particle term

$$\mathcal{L}_{(SM,1)} = \sum_{\text{gauge bosons}} -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_{\text{fermions}} \bar{\psi}\,\not{D}\psi + (D_\mu\Phi)^+(D^\mu\Phi) - V(\Phi)$$

Gauge boson interaction terms

Higgs field Φ terms

(to give mass)

$$V(\Phi) = -\mu^2\Phi^+\Phi + \lambda\left|\Phi^+\Phi\right|^2$$

SM fix-ups

W, Z masses
    - Approach yields deeper insight

Fermion masses
Quark mixing
CP violation
Neutrino mixing
(Neutrino CP violation?)
    - No deeper insight yet

ie. observed, unexplained properties of nature that must be added to the SM by hand.

W, Z Masses

- Everything in SM lagrangian is **massless**
- Mass conferred via **Higgs mechanism**
  - Introduce Higgs field (complex doublet) Φ = (0, v+H)
  - Subst. into lagrangian
  - $D_\mu \Phi D^\mu \Phi$ contains terms of the form

$$(g^2 v^2/4) W_\mu^+ W_\mu^- + (v^2/8)(g W_\mu^3 - g' B_\mu)^2$$

$$W_\mu^+ = (W_\mu^1 + i W_\mu^2)/\sqrt{2} \text{ etc.}$$

v = vacuum expectation value = $\mu/\sqrt{\lambda}$

⇒ Masses for gauge bosons + gauge invariance.

Mixing of U(1) and SU(2) gauge bosons (electroweak unification)

Prediction of Higgs boson (**m_H**)

Electroweak unification

W_μ^1, W_μ^2 mix after Higgs breaking → massive W^+ and W^-

$W^+ = (W_\mu^1 + W_\mu^2)/\sqrt{2}$ (massive)

$W^- = (W_\mu^1 - W_\mu^2)/\sqrt{2}$ (massive)

W_μ^3, B_μ mix after Higgs breaking → massive Z^0, γ

$Z^0 = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu$ (massive)

$\gamma = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3$ (massless)

Unification here!

SM relates M_W, M_Z and g', g:

$\tan\theta_W = g' / g$

$M_W = M_Z / \cos\theta_W$

Unification here!

Effect on couplings

Subst for W^+,W^-,Z in lagrangian;

- Couplings for **W±** of form (only l.h. fermions; $W_\mu{}^1$, $W_\mu{}^2$)

$$-g/2\sqrt{2}\ \bar{\nu}\gamma_\mu(1-\gamma^5)eW_\mu{}^-$$

- Couplings for **photons** of form

$$g\sin\theta_W \bar{e}\gamma_\mu e A_\mu$$

γ_μ transforms as vector (P odd)
$\gamma_\mu\gamma^5$ transforms as axial vector (P even)
(Vector + axial vector ⇒parity violation for weak force)

Effect on couplings

Subst for W^+,W^-,Z in lagrangian;

- Couplings for **Z^0** of form (l.h. (W_μ^3) and r.h. (B_μ) fermions)

$-g/4\cos\theta_W\,\bar{\nu}\gamma_\mu(1-\gamma^5)\nu Z_\mu$ (lh neutrinos)

$g/4\cos\theta_W\,\bar{e}(\gamma_\mu(1-\gamma^5)-4\sin^2\theta_W\gamma_\mu)eZ_\mu$ (lh & rh e)

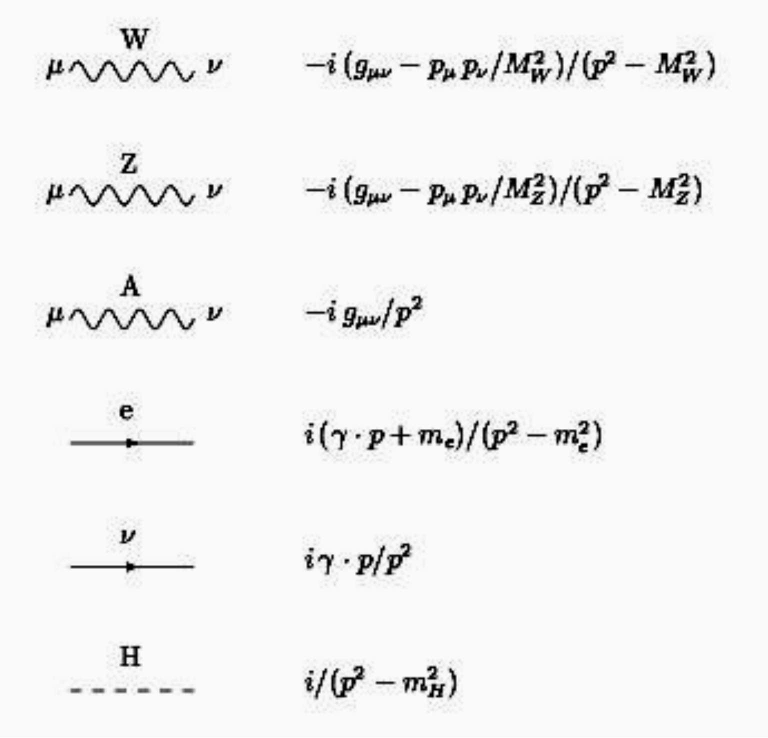
⇒ Effective vector and axial couplings g_{vf}, g_{af} for Z^0 decays;

$g/4\cos\theta_W\,\bar{e}(\gamma_\mu(g_{vf}-g_{af}\gamma^5))eZ_\mu$

fermion	g_{vf}	g_{af}
$\nu e,\nu\mu,\nu\tau$	0.5	0.5
e,μ,τ	$-0.5+2\sin\theta_W$	-0.5
u,c,t	$0.5-4/3\sin^2\theta_W$	0.5
d,s,b	$-0.5+2/3\sin^2\theta_W$	-0.5

Aside: Feynman rules

Propagators (free particle L)



$-i\,(g_{\mu\nu} - p_\mu p_\nu/M_W^2)/(p^2 - M_W^2)$

$-i\,(g_{\mu\nu} - p_\mu p_\nu/M_Z^2)/(p^2 - M_Z^2)$

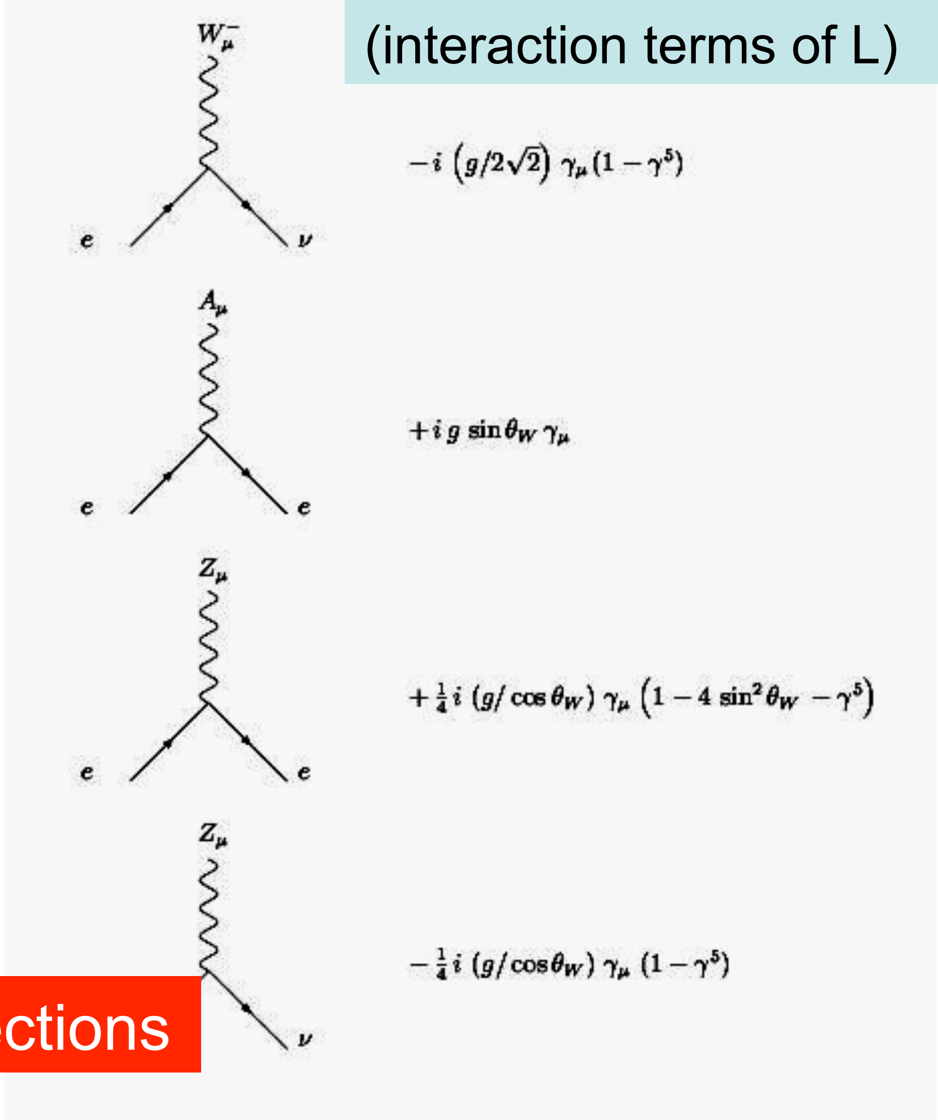
$-i\,g_{\mu\nu}/p^2$

$i(\gamma \cdot p + m_e)/(p^2 - m_e^2)$

$i\gamma \cdot p/p^2$

$i/(p^2 - m_H^2)$

vertex factors (interaction terms of L)



$-i\left(g/2\sqrt{2}\right)\gamma_\mu(1-\gamma^5)$

$+i\,g\,\sin\theta_W\,\gamma_\mu$

$+\frac{1}{4}i\,(g/\cos\theta_W)\,\gamma_\mu\left(1-4\sin^2\theta_W-\gamma^5\right)$

$-\frac{1}{4}i\,(g/\cos\theta_W)\,\gamma_\mu\,(1-\gamma^5)$

Allow us to calculate cross-sections

Fermion masses

- Assumed to arise from Higgs
- Simplest (Yukawa) coupling **assumed**

$$g\varphi_0\bar{\psi}\psi$$

g,ϕ_0 are (unknown) constants (looks like mass term)

Form proposed by Glashow.

Can now be tested.

Weak and strong eigenstates

Quarks interact through the strong force

Gluons couple to strong (physical) quark eigenstates q

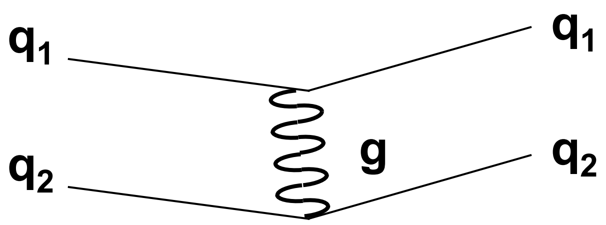
→ hadrons

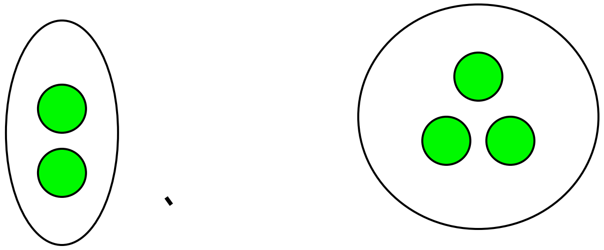
Quarks also interact through the weak force

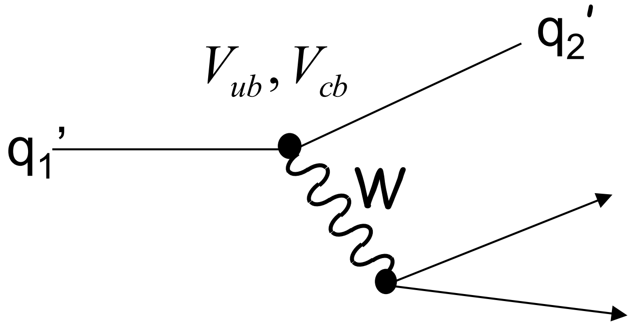
W couples to weak quark eigenstates q’

q’ related to q through mixing matrix

q’ admixture of q and v.v.







Weak, strong eigenstates

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

Strong quark eigenstates

Gluons couple to strong quark eigenstates → observable hadrons

$$\begin{pmatrix} u \\ d_W \end{pmatrix} \quad \begin{pmatrix} c \\ s_W \end{pmatrix} \quad \begin{pmatrix} t \\ b_W \end{pmatrix}$$

Weak quark eigenstates

W boson couples to weak quark eigenstates

(**convention** to change bottom member of family)

Weak quark eigenstates are admixtures of strong quark eigenstates

Quark mixing

$$\begin{pmatrix} d_W \\ s_W \\ b_W \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Strong, weak eigenstates related by mixing matrix

Mixing matrix is unitary (inverse = complex conjugate)

CKM matrix

3x3 matrix = CKM matrix (1973 – before charm!)

Elements describe every weak quark transition

SM does not predict existence of or values for matrix elements (couplings of W to quarks).

Input by experimental data

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CP violation

C = charge operator
P = parity operator

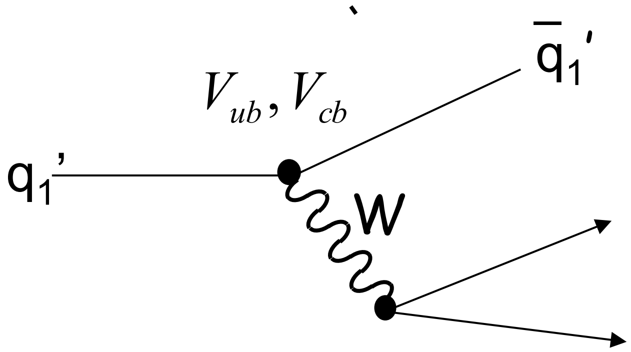
CP operation changes particle to antiparticle (and vice versa)
CP **violation** if part.->anti part. rate different to anti part. -> part.

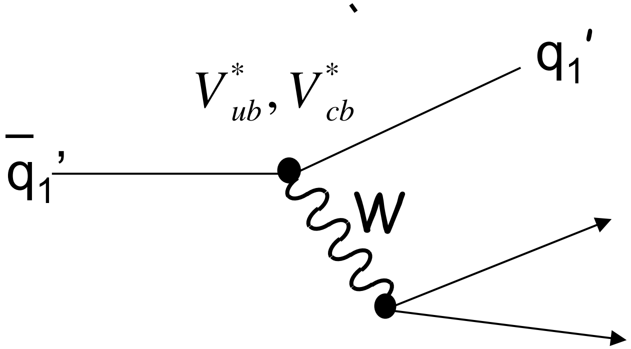
CP violation observed in weak decays.

Must be added to Standard Model.
($V_{ub} \neq V_{ub}^*$ etc.)

Note:

- **SM does not predict** CP violation.
- **SM does not explain** CP violation.





CP violation

- Need 3 generations of quarks to introduce CP violation into theory

$$\begin{pmatrix} d_W \\ s_W \\ b_W \end{pmatrix} = \begin{pmatrix} \; & \; & \; \\ \; & \; & \; \\ \; & \; & \; \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Mixing matrix is 3x3.

Unitarity constraints ⇒ 4 independent parameters

3 angles quantify mixing between (1,3) (2,3) (1,2) generations,
1 complex phase (mechanism for introducing CP)

Neutrino CP violation, mixing

- Similar framework adopted for neutrinos (MNS matrix). Flavour (e, μ, τ) related to mass eigenstates:

$$\begin{pmatrix} v_e \\ v_\mu \\ v_\tau \end{pmatrix} = \begin{pmatrix} \quad & \quad & \quad \\ & & \\ & & \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

Mixing matrix is 3x3.

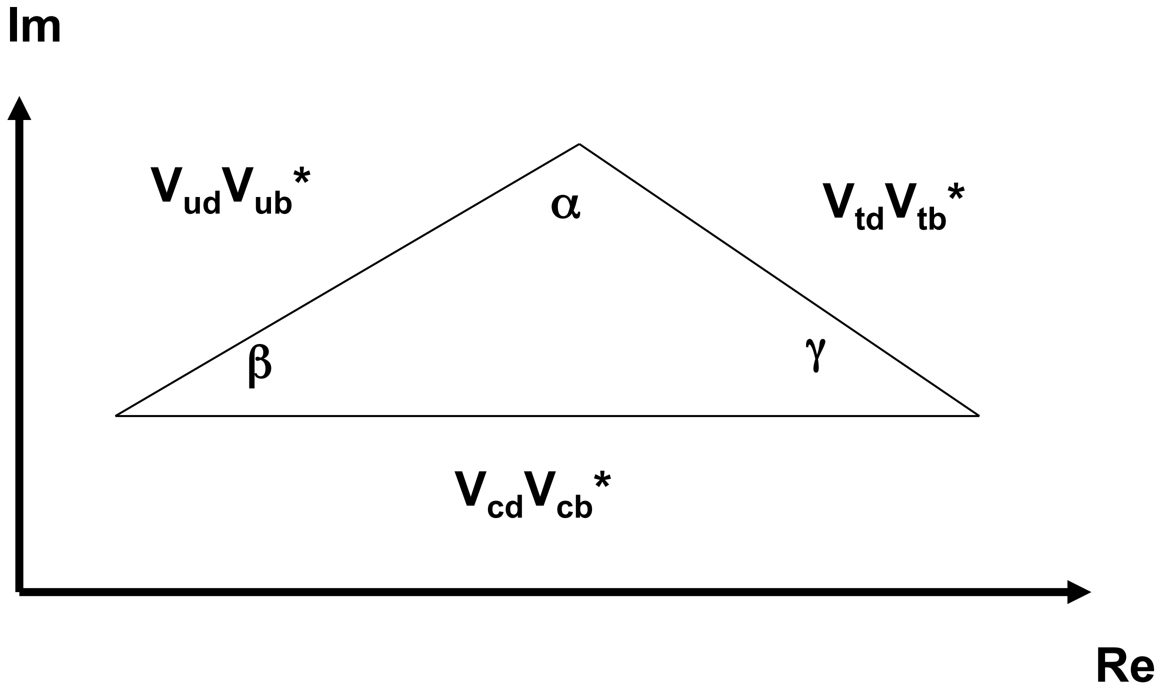
Unitarity constraints ⇒ 4 independent parameters

3 angles quantify mixing between (1,3) (2,3) (1,2) generations,
1 complex phase (mechanism for introducing CP)

Testing: Unitarity triangle

SM constraint of unitarity gives relations between unknown CKM matrix elements, eg:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



$$\alpha = \arg\left\{-\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}}\right\}$$

$$\beta = \pi - \arg\left\{\frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}}\right\}$$

$$\gamma = \arg\left\{-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right\}$$

How many free parameters?

SM does not predict:

- Magnitude of gauge couplings g, g', g_s
- Masses of fermions (3 leptons, 6 quarks, 3 neutrinos)
- Weak-strong eigenstate quark mixing (can express by 4 parameters), ditto for neutrinos
- Higgs related quantities (mass of Higgs and vacuum expectation value)

⇒ **Some 26 unknowns in the theory.**

- These values must be added by hand (experimental measurements)

SM predicts relationships

- All observables can be predicted in terms of 26 free parameters
  - If we have > 26 measurements of these observables, we overconstrain SM
  - Overconstrain ⇒ we don’t have any more ad hoc inputs AND we can test the consistency of the model
- **Best plan:**
  - pick well measured set of observables
  - Calculate other observables in terms of these well known quantities
  - Test predictions; measure observable, compare to theory

Testing the SM

1. Test any assumptions we’ve made
2. Measure unknown parameters in different ways and check consistency
3. Compare predicted quantities to measurements
4. Check internal consistency of entire SM framework

Where can the SM be tested?

Particle physics experiments designed to test specific aspects of SM

- Major historical experiments:
  - LEP (ALEPH,DELPHI,L3,OPAL) $(\sqrt{s} = M_Z \rightarrow >2M_W)$ **Electroweak, qcd**
  - Babar, Belle $(\sqrt{s} = 2M_B)$ **Quark mixing, CP**
  - CDF, D0 $(\sqrt{s} = 2$ TeV$)$ **electroweak, qcd, quark mixing**
  - H1, Zeus $(\sqrt{s} = 0.95$ TeV$)$ **qcd**
- Major running experiments:
  - ATLAS,CMS,LHCb $(\sqrt{s} = 7{,}8$ TeV$)$

electroweak, qcd, quark mixing, CP

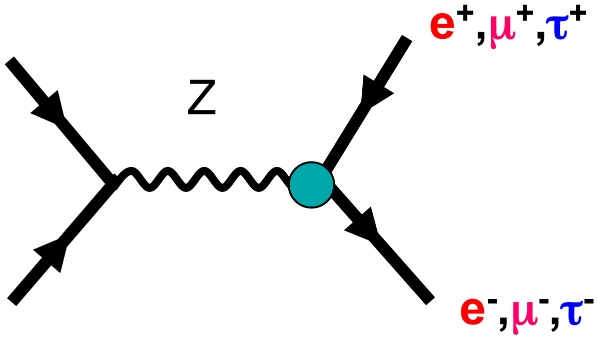
Assumption: Lepton universality

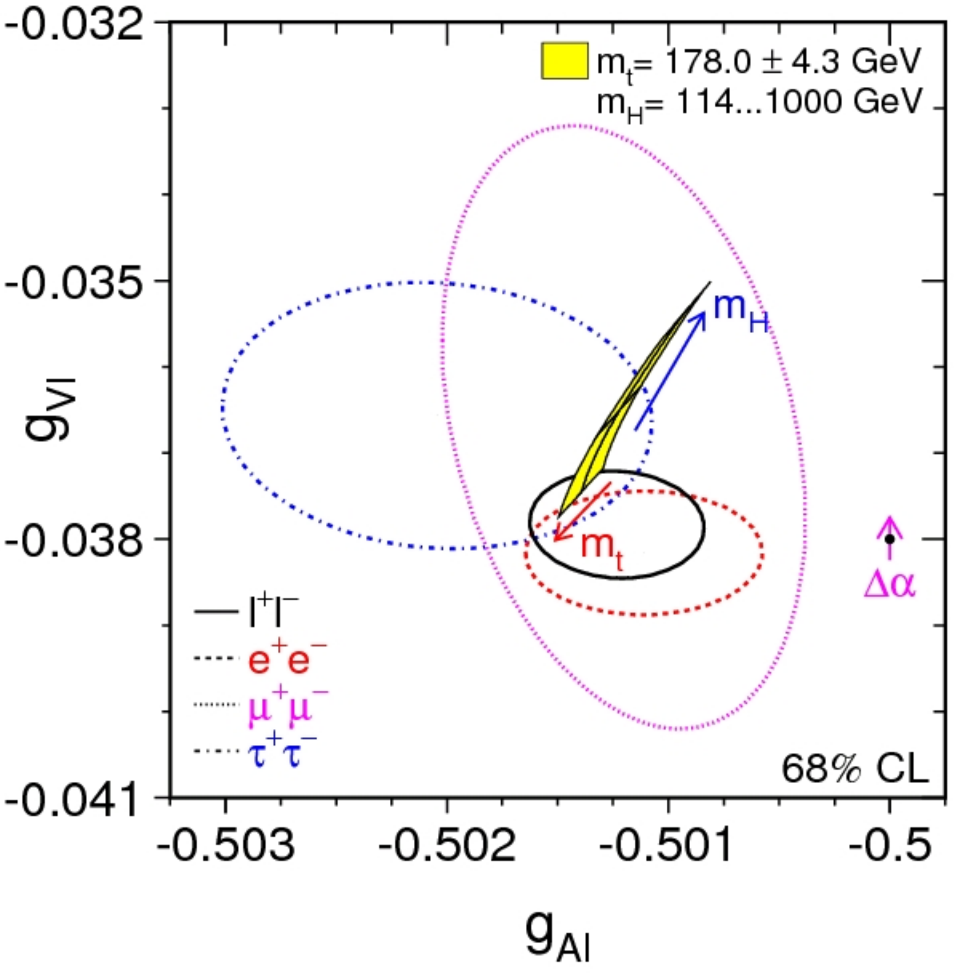
Assumed in SM that e, μ, τ have similar ewk couplings

Test Z couplings to ee μμ ττ

Find all measurements consistent with each other

Also consistent with SM prediction





Assumption: 3 generations of matter

No info on # generations in SM

Use Z lineshape: SM relates width to possible decay products

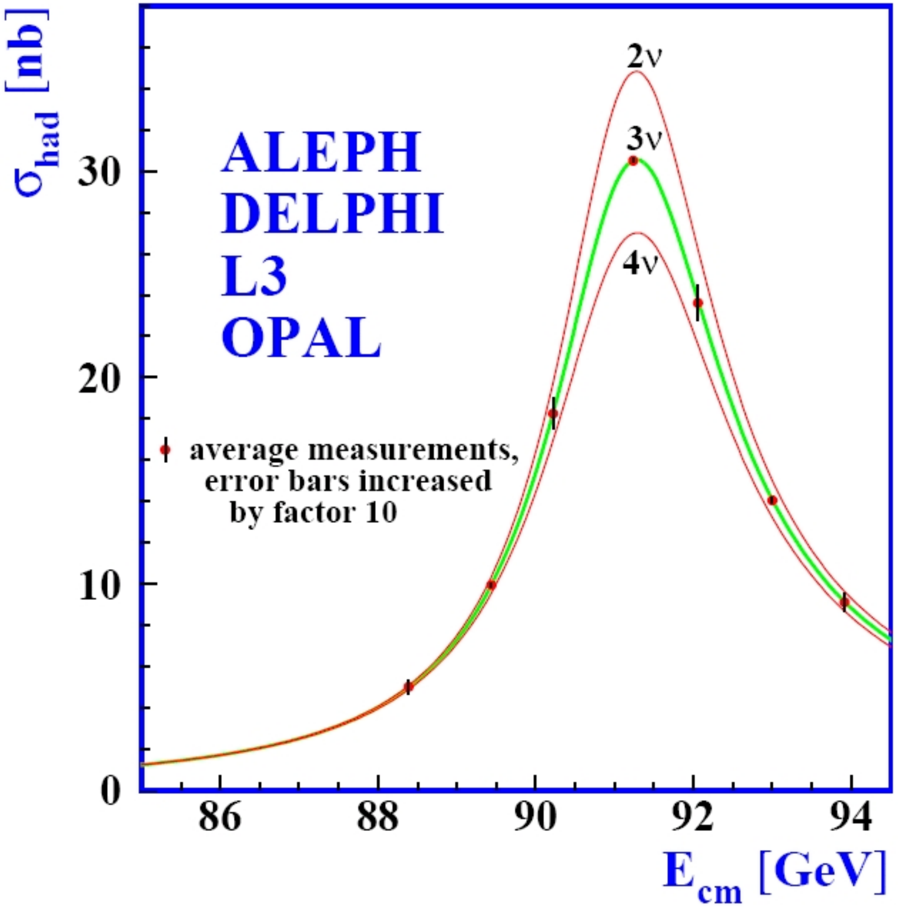
Measure Γ_{had}, Γ_{ee}, σ_{had}

σ_{had} is a function of Γ_Z, Γ_{had}, Γ_{ee}

$\Gamma_Z = \Gamma_{had} + \Gamma_{ll} + \Gamma_{inv}$

Assume $\Gamma_{inv} = N\nu\Gamma_{\nu\nu}$

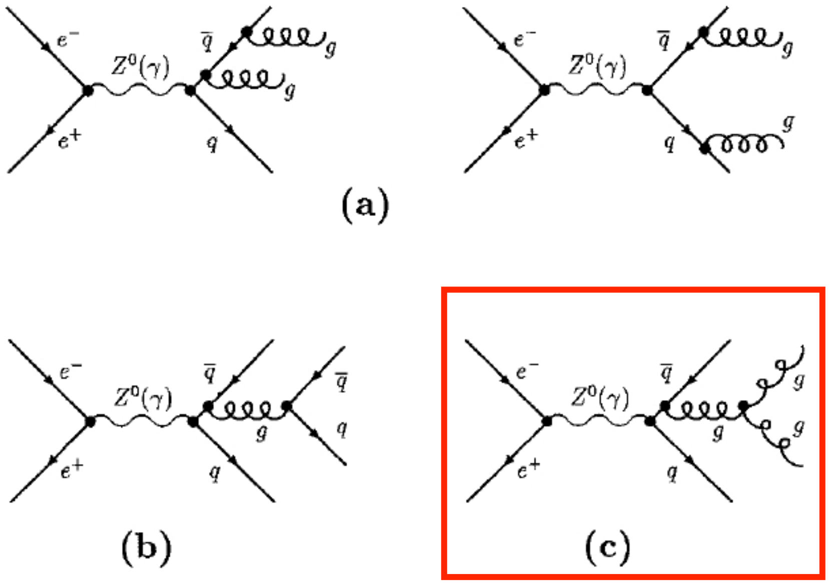
Calculate $N\nu$



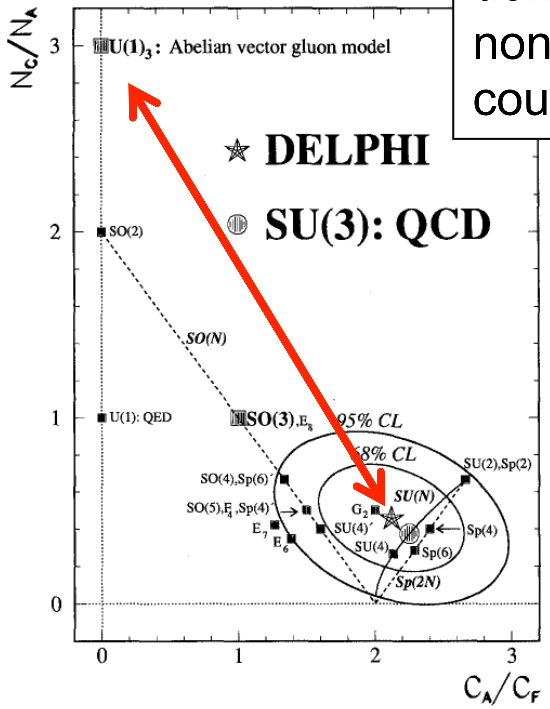
$N\nu = 2.984 \pm 0.008$

Test: Non-abelian strong force

Rate of 4 jet production at LEP:



SM contribution from **triple gluon coupling**

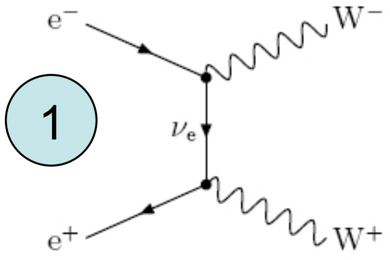


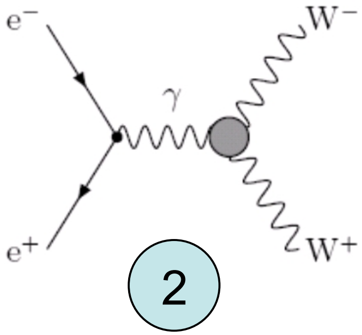
Difference demonstrates non-abelian couplings

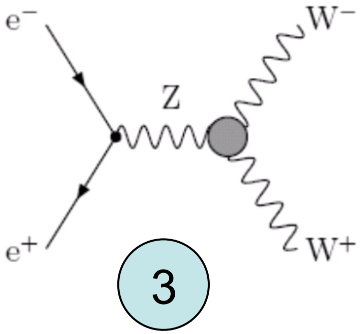
Fig. 10. 68% and 95% CL contour plots for the measured variables C_A/C_F and N_C/N_A, and expections from different gauge theories. C_A/C_F = ratio of coupling strength of $g \to gg$ to $q \to qg$; N_C/N_A = number of quark colours divided by the number of gluons.

Zeit. Phys. C59 (3) (1993) 357

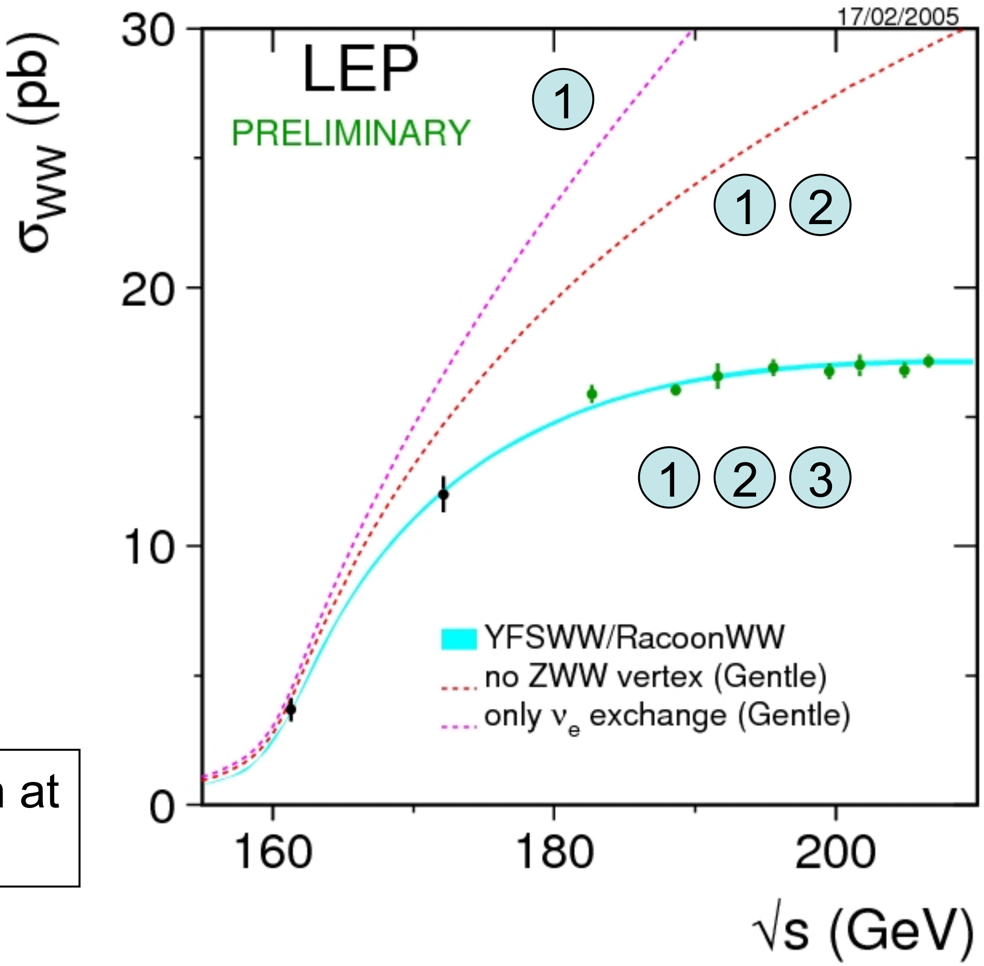
Test: Non-abelian weak force







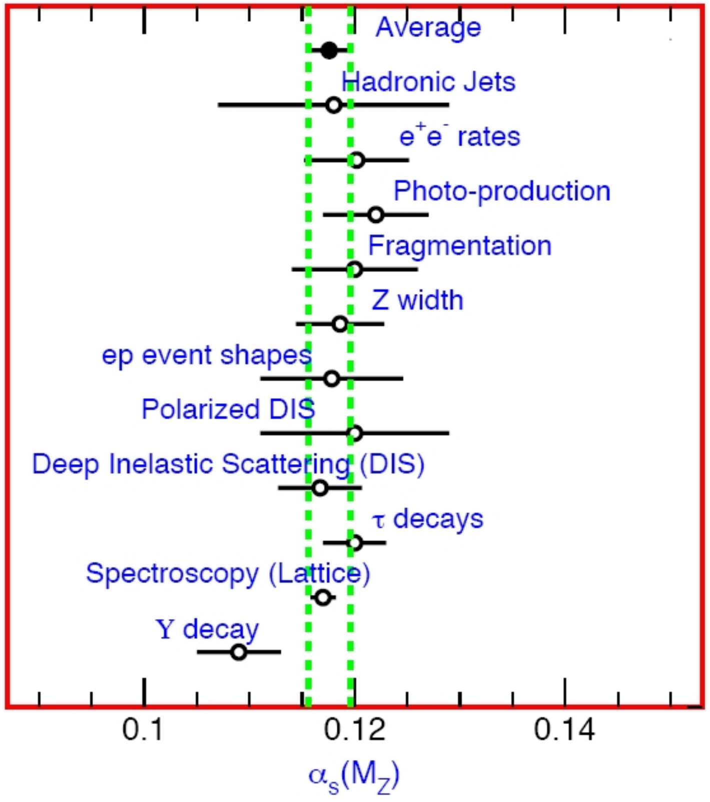
Check rate of $e^+e^- \to$ WW production at LEP



LEP 4fermion working group

Input: Couplings

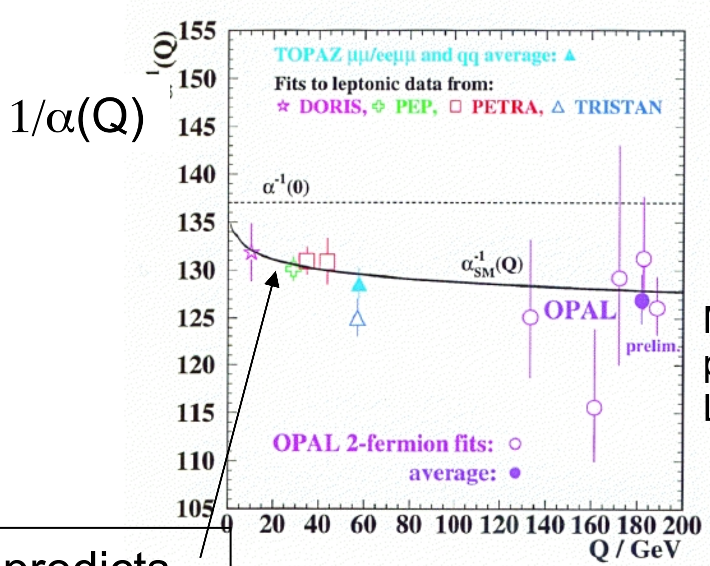
Strong coupling



Many measurements give consistent answer

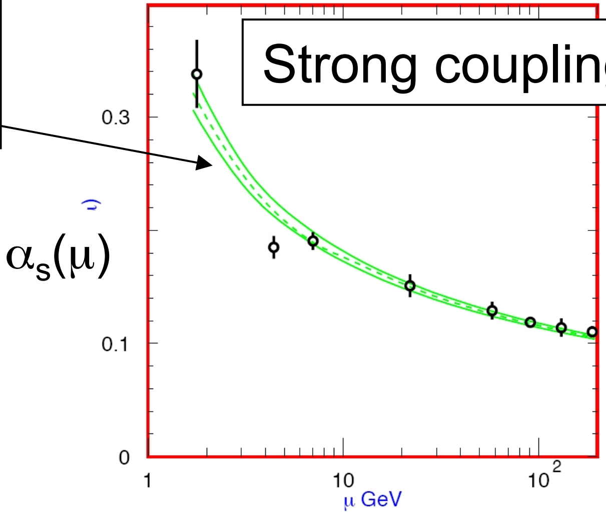
Particle data group

EM coupling



M. Kobel, proceedings LP97

SM predicts evolution with energy

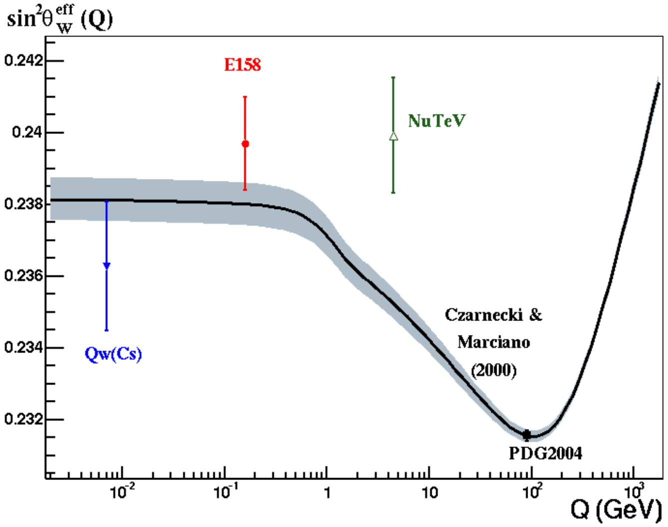


Strong coupling

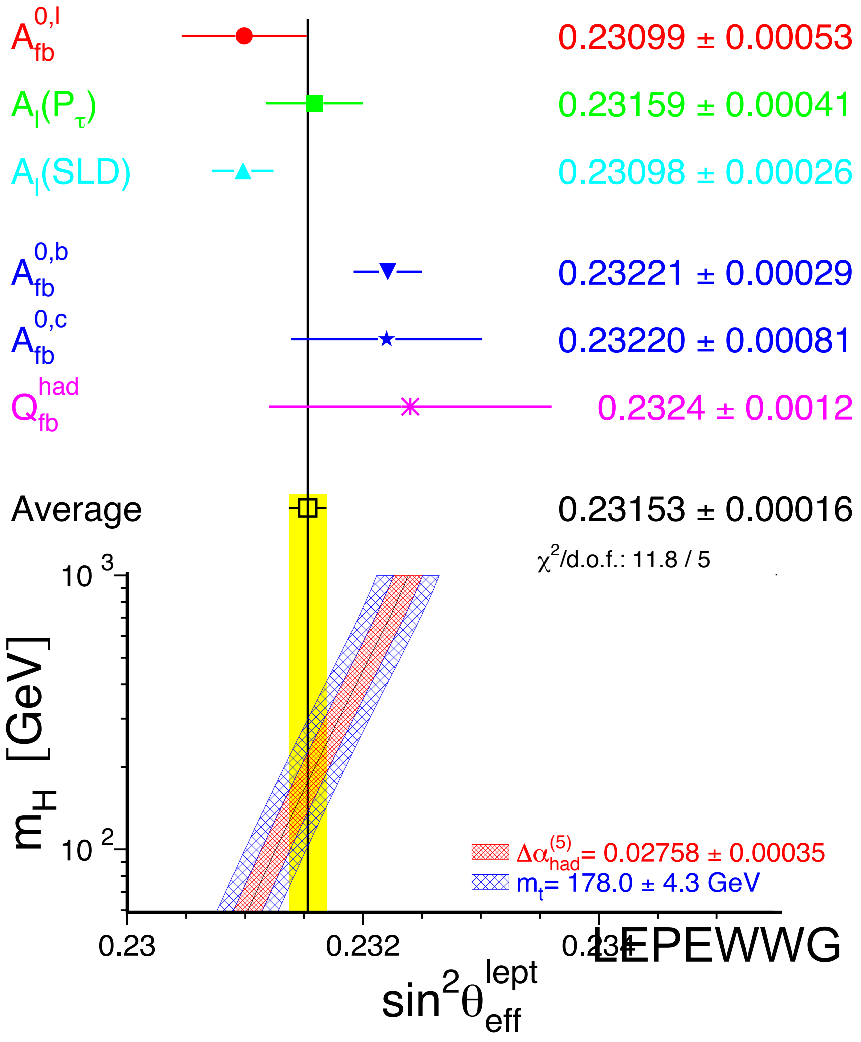
Input: $\sin^2\theta_W$

Relates weak, em couplings and M_W, M_Z

Consistent result extracted from many different measurements

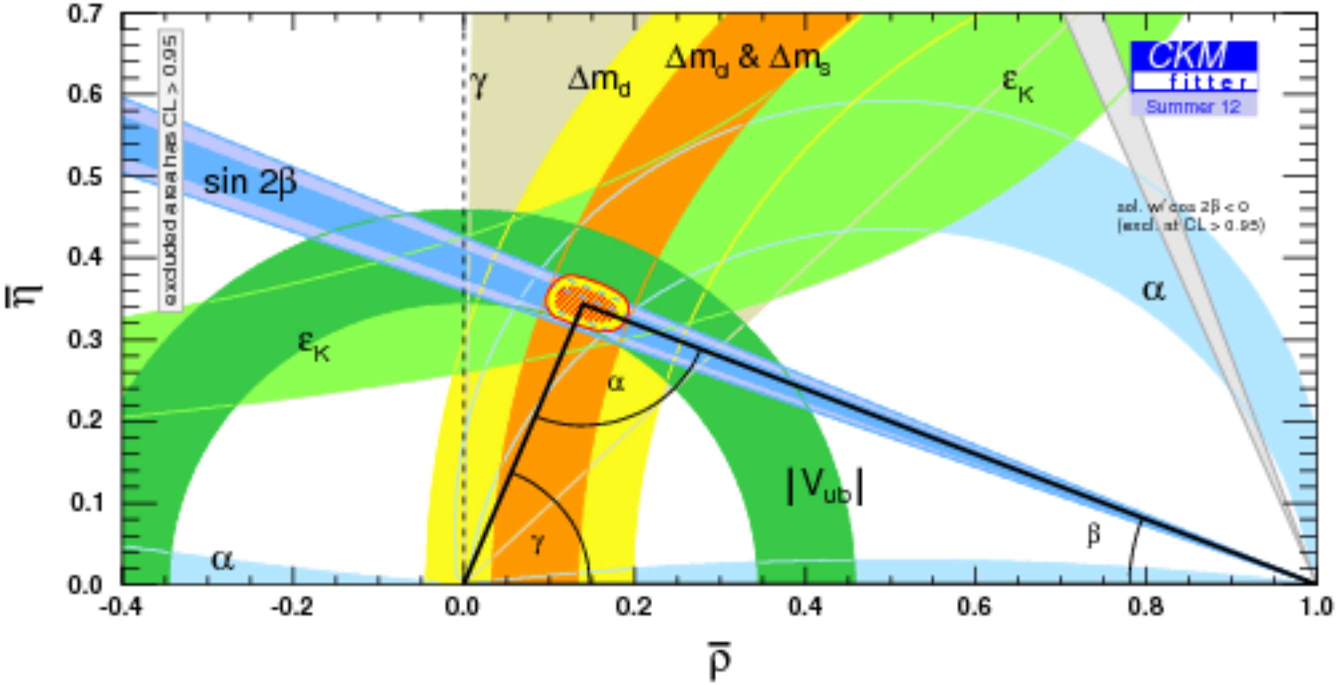


Energy evolution



Input: Quark mixing, CP

Many measurements of the 4 parameters describing quark mixing and CP violation



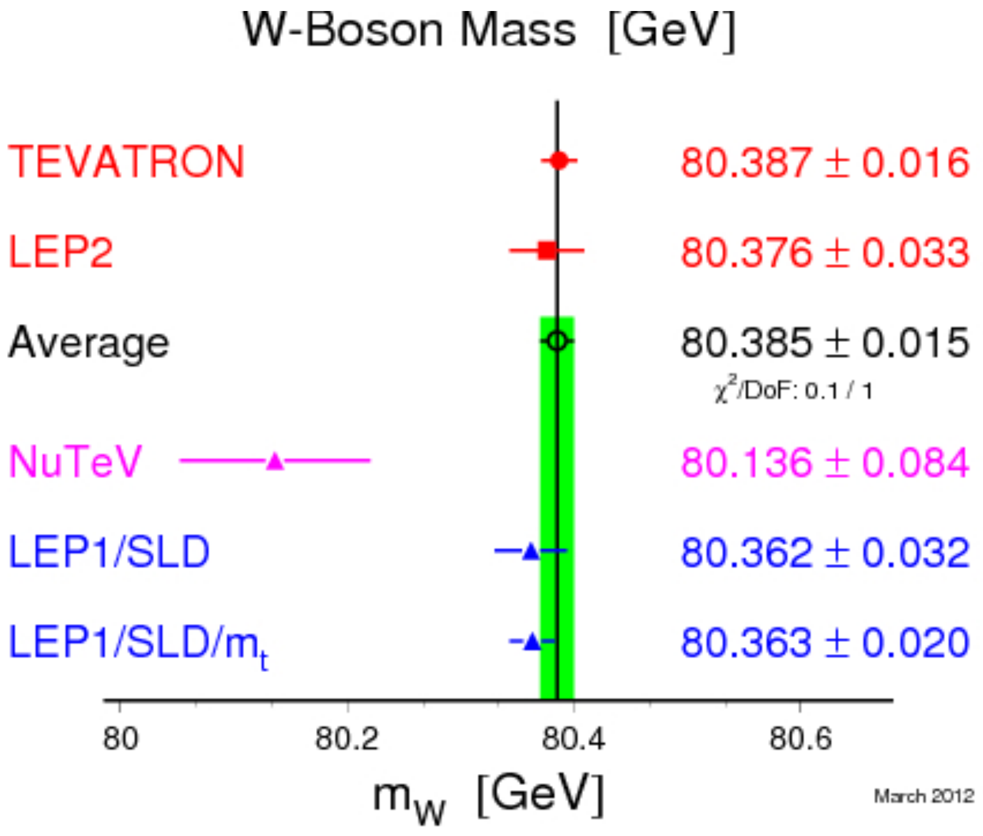
All measurements consistent

Input/test: W boson mass

Direct measurement

Inferred (NuTeV $\sin^2\theta_W + M_Z$)

SM prediction

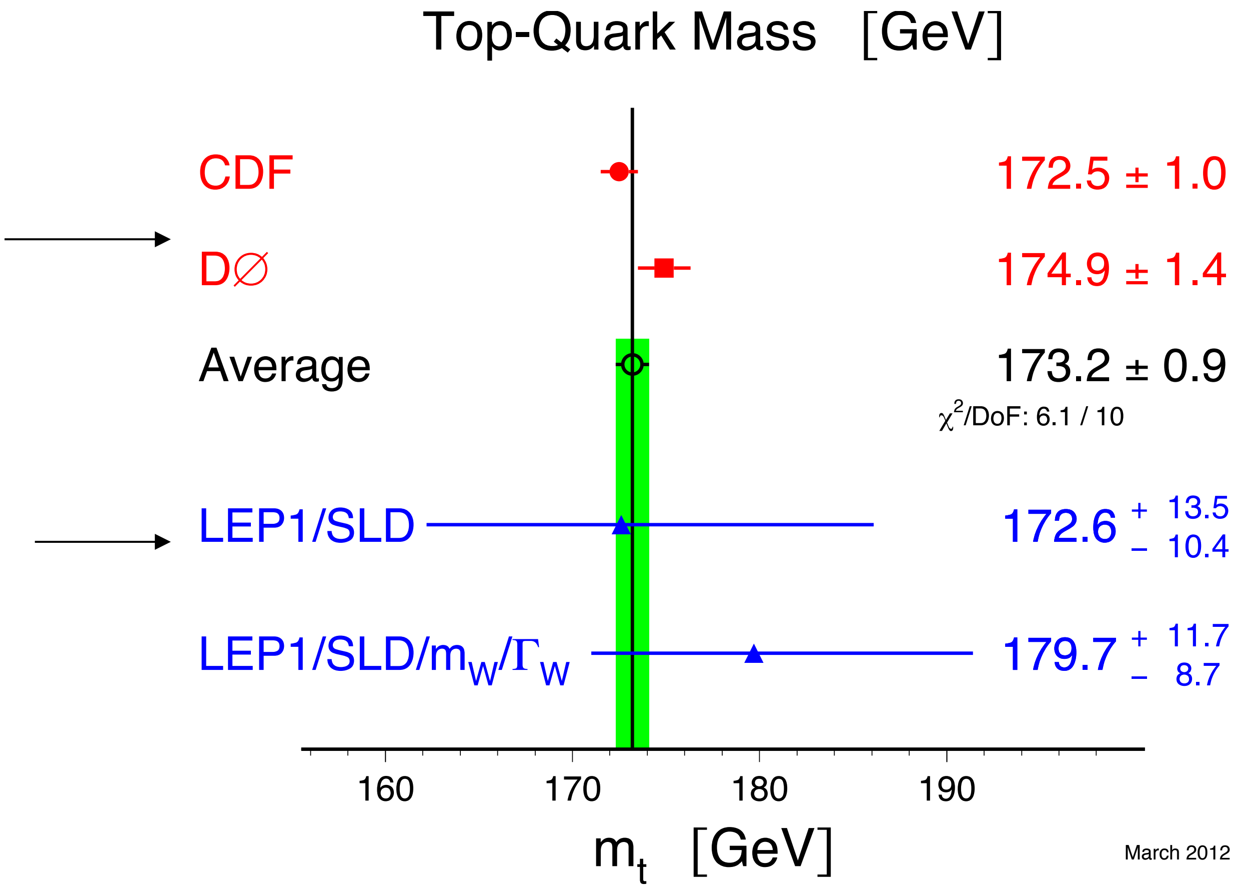


Consistent (NuTeV result low)

Input/test: Top quark mass

Direct measurement

SM prediction

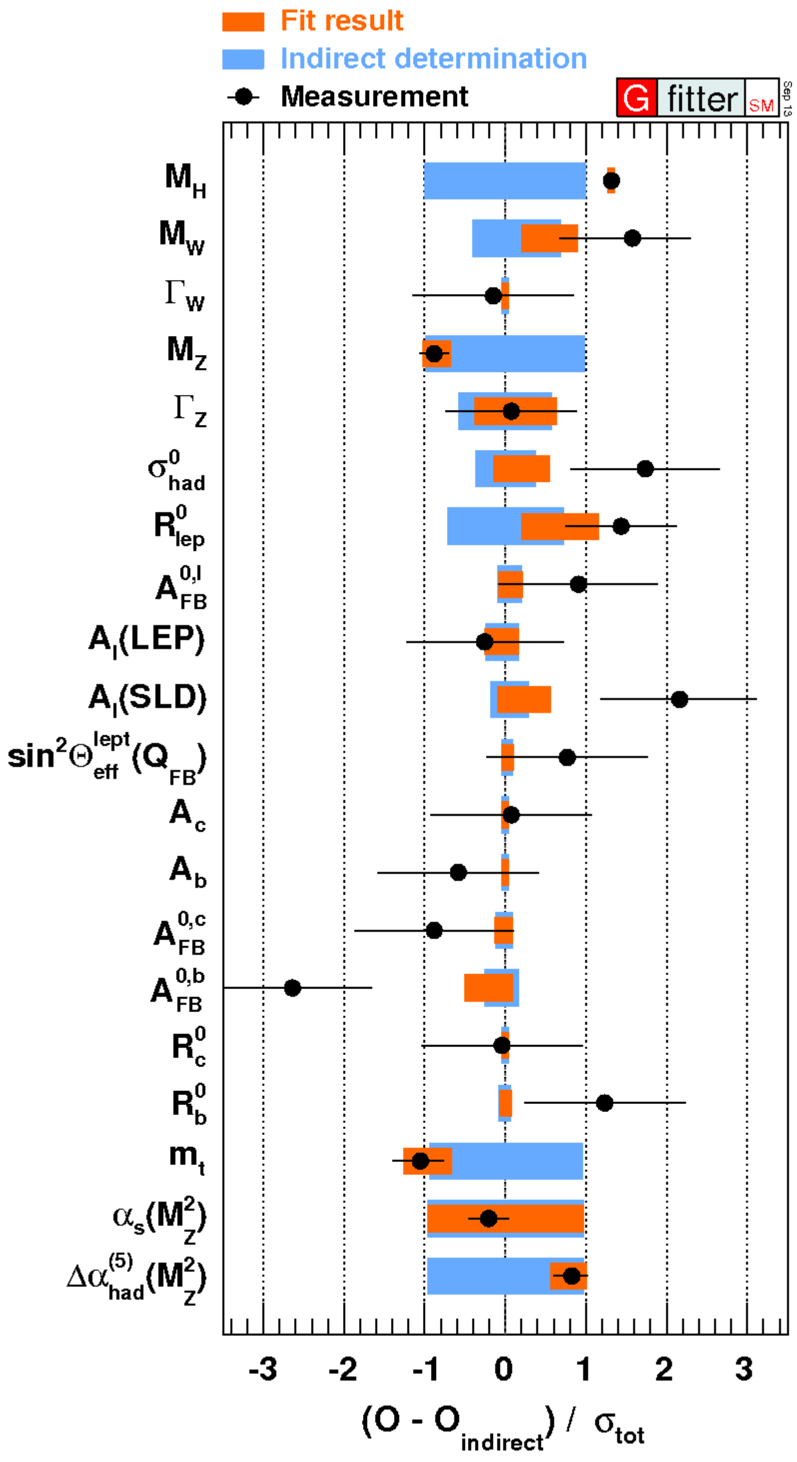


Consistent

Internal consistency

Many consistency checks possible

- Here fit to Z, W top quark results is shown
- χ^2/ndf = 18.2/13 (Probability ~15 %)



Shortcomings

1. **Experimental**
    - Still haven’t experimentally verified all of SM
    - Any differences wrt predictions could signal New Physics
2. **Philosophical**
    - There is a lot we still don’t understand
    - What lies beyond the limits of the SM?

Experimental disagreements?

NuTeV $\sin^2\theta_W$ (~ 3 σ)

Extract from ratio of neutral:charged ν nucleon couplings

New Physics? (eg. Z’, new fermions)

Or analysis? (uncertainties in pdfs, radiative corrections)

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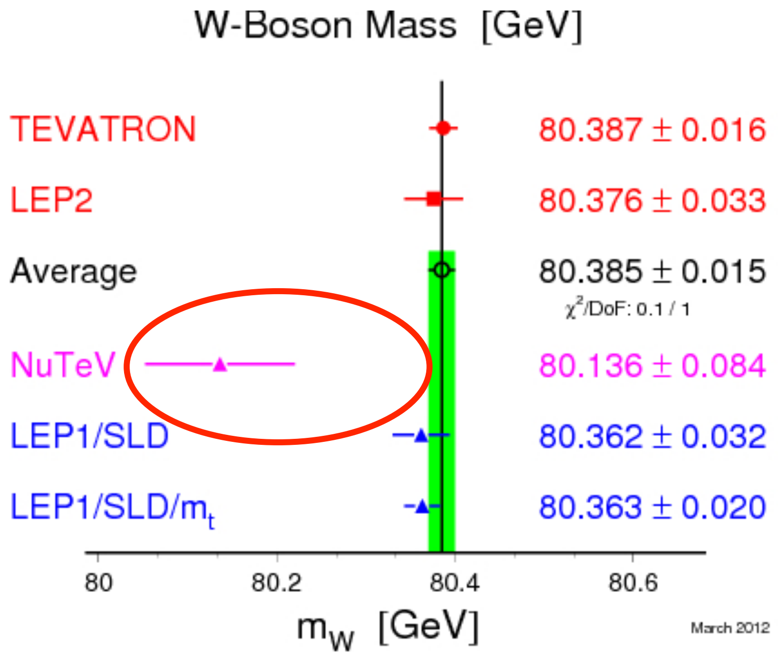

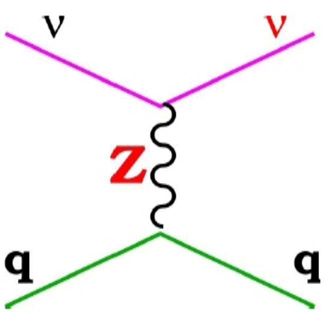

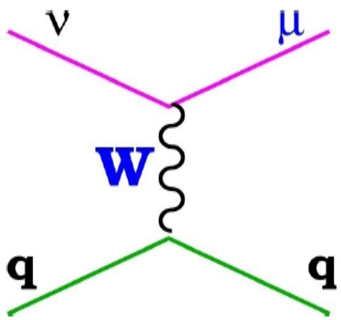

Experimental disagreements?

Couplings of b quarks?

A_{fb} measured vs. SM prediction (2.8 σ)

No identified experimental explanation

Assumed to be a fluctuation

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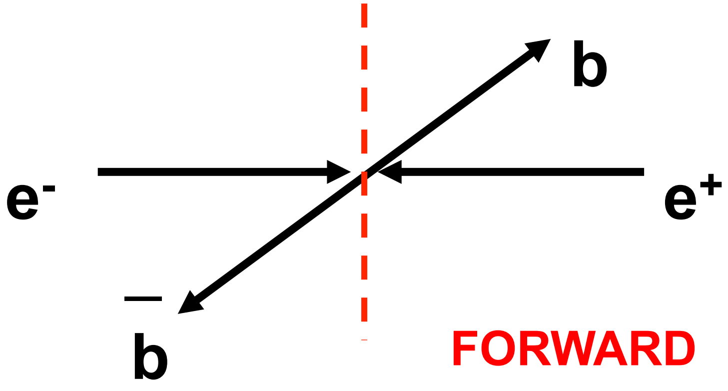
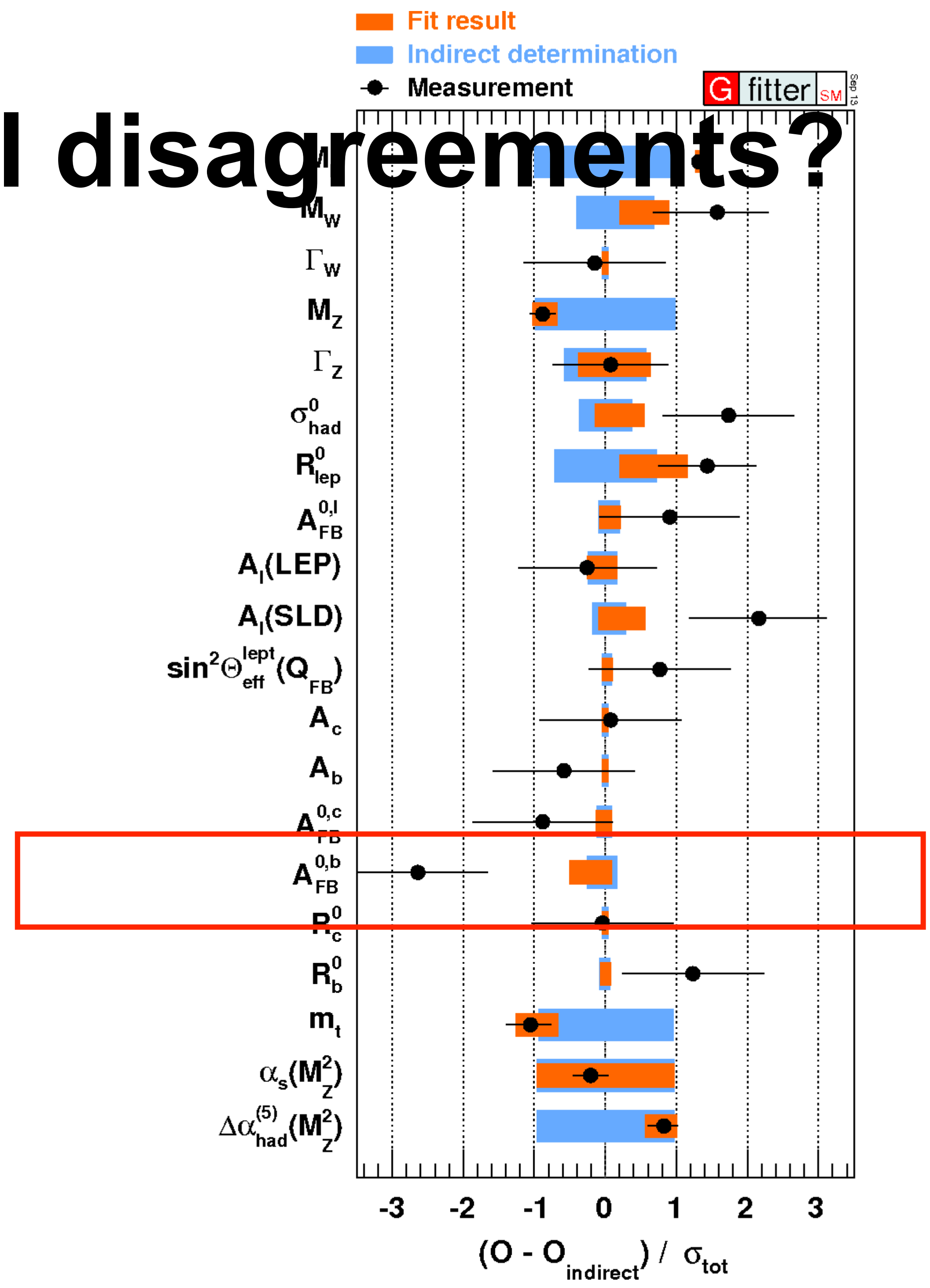
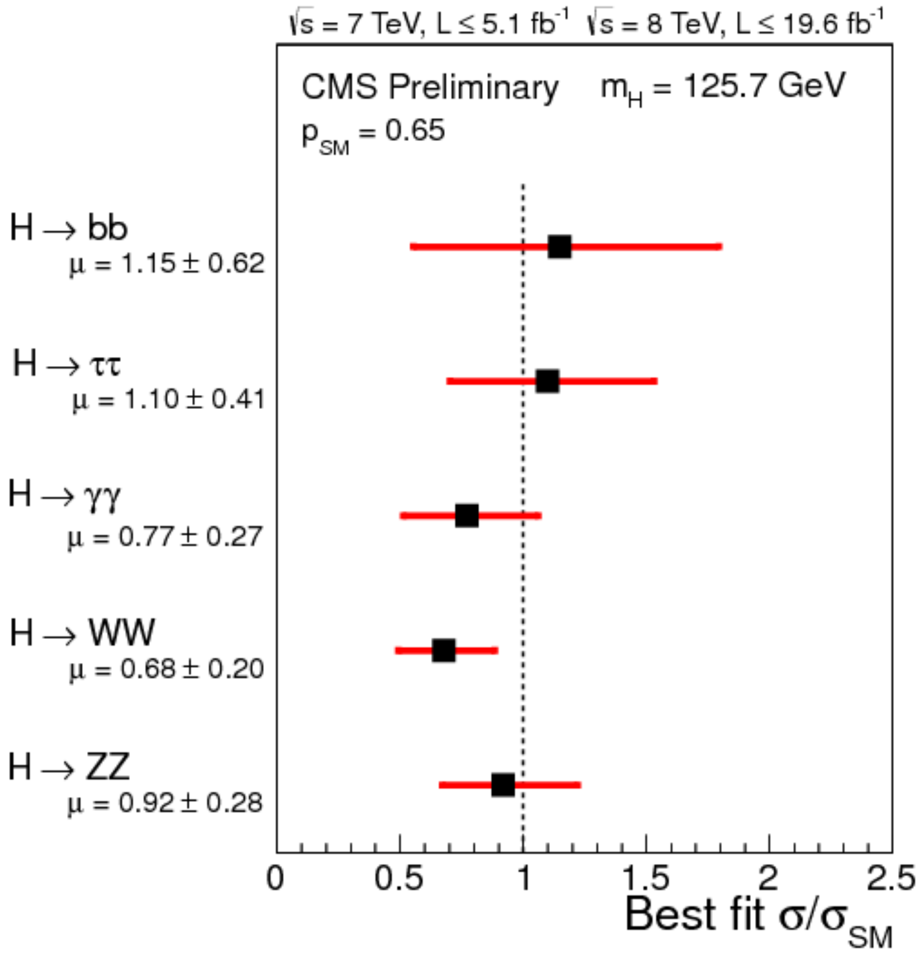
A_{fb} = (Nf – Nb)/(Nf+Nb)





Might be an issue…

Is it a Higgs?
A Standard Model Higgs?



(updates from 2015…)

What else is wrong?

Fine tuning:

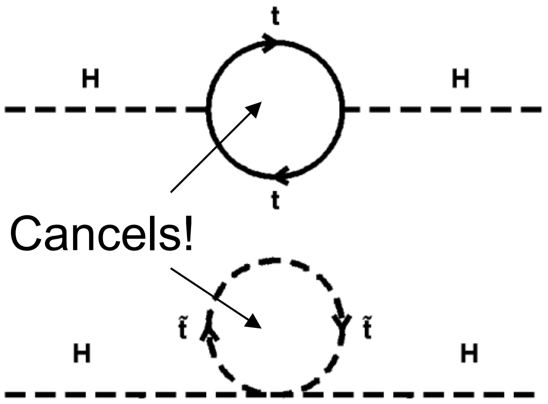
Higgs mass calculation: bare mass + radiative corrections + loops

very careful choice of parameters to ensure mass ~ 100 GeV

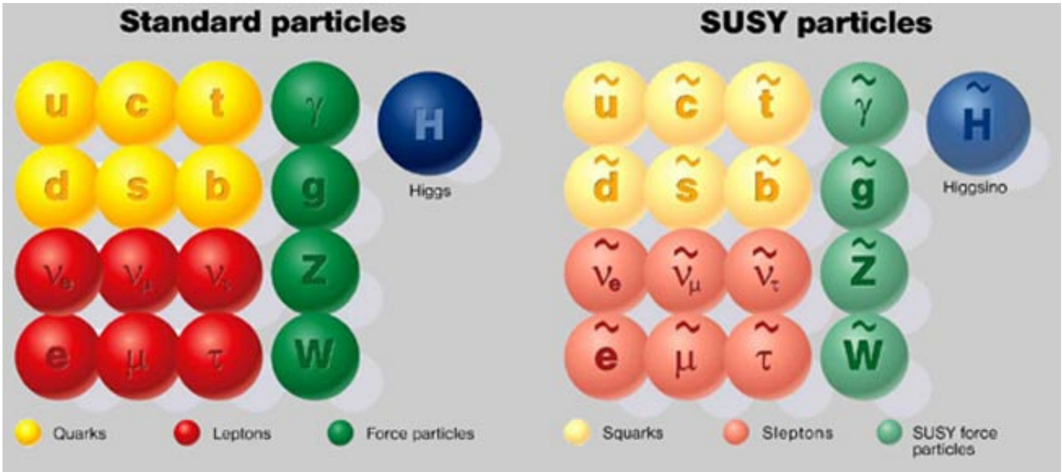
Supersymmetry:

Symmetry between fermions and bosons

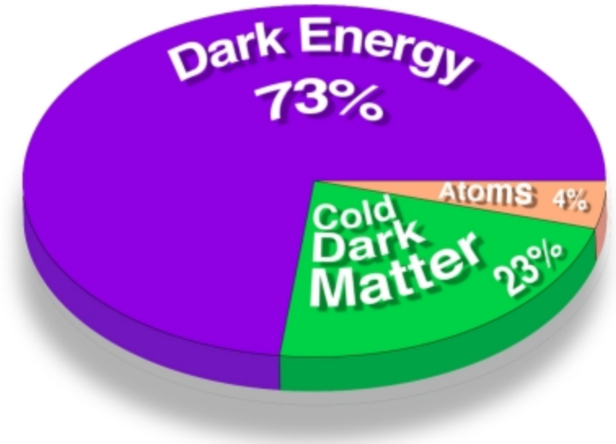
New supersymmetric particles can cancel divergences in mH calculation



Cancels!



4% of the universe?



Source: Robert Kirshner
Source: NASA/WMAP Science Team

SM with electroweak and strong interactions only describes 4% of the universe

Dark energy:

?

Dark matter?

Try Supersymmetry ….

Lightest supersymmetric particle is a dark matter candidate (massive and unobservable)

And finally ….

Gravity

Can’t describe it in SM

Can include it in string theory – not very testable

Large extra dimensions could be observed at full LHC energy .. perhaps.

CP violation

Consistent picture in SM but insufficient to explain matter – antimatter asymmetry of the universe

? Answer lies in new physics?

Many open questions not addressed by the SM

Review

SM unites electromagnetic, weak, strong forces

SM predicts cross-sections, couplings

SM incomplete – 26 free parameters

- Relations between some free parameters are predicted
- So far theory is consistent with experimental findings, if incomplete.