Standard Model

Quark mixing and CP violation

Overview

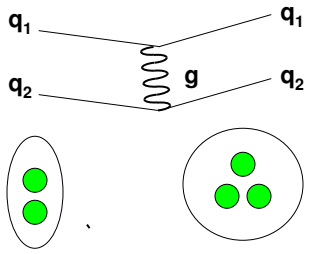
- Quark weak and strong eigenstates
- 2 generations
  - review K^0 mixing
- 3 generations
  - review K^0 mixing, B^0 mixing
- CP violation
  - Alternative parametrisations of CKM matrix

Weak and strong eigenstates

Quarks interact through the strong force

Gluons couple to strong (physical) quark eigenstates q

→ hadrons

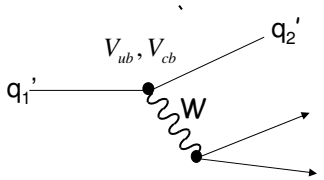


Quarks also interact through the weak force

W couples to weak quark eigenstates q'

q' related to q through mixing matrix

q' admixture of q and v.v.



Weak, strong eigenstates

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

Strong quark eigenstates

Gluons couple to strong quark eigenstates → observable hadrons

$$\begin{pmatrix} u \\ d_W \end{pmatrix} \quad \begin{pmatrix} c \\ s_W \end{pmatrix} \quad \begin{pmatrix} t \\ b_W \end{pmatrix}$$

Weak quark eigenstates

W boson couples to weak quark eigenstates

(**convention** to change bottom member of family)

Weak quark eigenstates are admixtures of strong quark eigenstates

Weak, strong eigenstates

$$\begin{pmatrix} d_W \\ s_W \\ b_W \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Strong, weak eigenstates related by mixing matrix

Mixing matrix is unitary (inverse = complex conjugate)

Two quark families

$$\begin{pmatrix} d_W \\ s_W \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Mixing matrix is 2x2

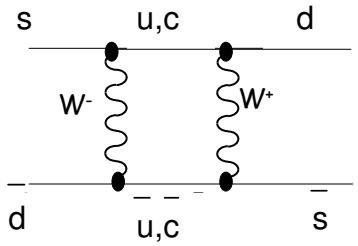
Condition of unitarity $\Rightarrow$ 1 unknown parameter

Usually taken as Cabbibo angle θ_c – quantifies mixing between two generations

Value not predicted by SM!

Two generations (K^0 mixing)

Explains K^0-$\bar{K}^0$ mixing:



W couples to d_W,s_W

Amplitudes for transitions proportional to Cabbibo angle and quark mass

CKM matrix

3x3 matrix = CKM matrix (1973 – before charm!)

Elements describe every weak quark transition

SM does not predict existence of or values for matrix elements (couplings of W to quarks).

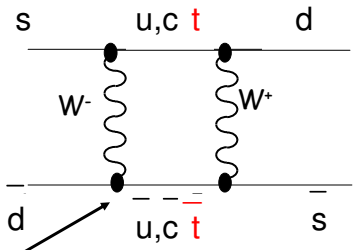
Input by experimental data

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Effect on K^0 mixing

Mixing:

Extra contribution from top due to small b_W admixture



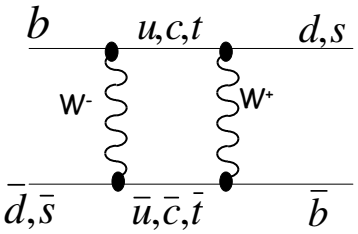
Couplings proportional to quark mass and CKM matrix element

Here charm CKM elements dominate

Amplitude $f(m_q)$

Box amplitude $\alpha \sum_{i=u,c,t} \sum_{j=u,c,t} V_{is} V_{id}^* V_{js}^* V_{jd} a_{ij}$

B^0 mixing



Mixing: expect B^0_d and B^0_s systems to mix like K^0

Now top CKM matrix elements dominate

B^0_s mixing faster than B^0_d as $V_{ts} > V_{td}$

Amplitude $\alpha \sum_{i=u,c,t} \sum_{j=u,c,t} V_{ib} V_{id}^* V_{jb}^* V_{jd} a_{ij}$

V_{tb}

Mass eigenstates not the same as weak eigenstates

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Different measurements sensitive to these

$$\begin{pmatrix} 0.9742 \text{ to } 0.9757 & 0.219 \text{ to } 0.226 & 0.002 \text{ to } 0.005 \\ 0.219 \text{ to } 0.225 & 0.9734 \text{ to } 0.9749 & 0.037 \text{ to } 0.043 \\ 0.004 \text{ to } 0.014 & 0.035 \text{ to } 0.043 & 0.9990 \text{ to } 0.9993 \end{pmatrix}$$

K^0 mixing sensitive

B^0 mixing sensitive

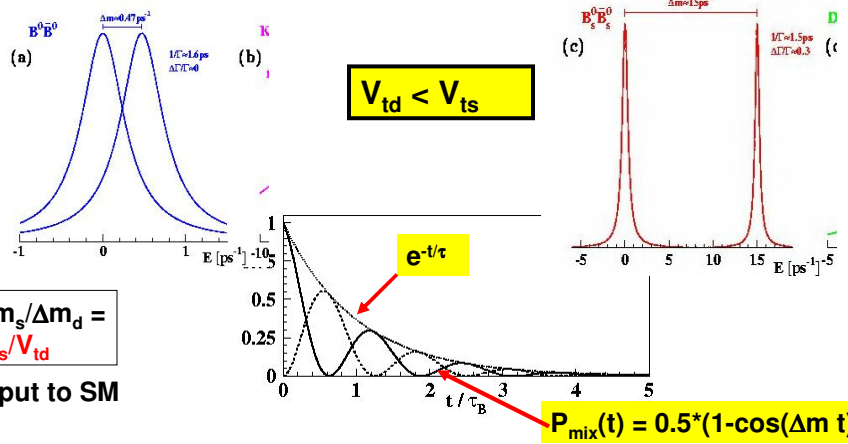
B^0 meson mixing

B^0_d:

– $\Delta m_d = 0.489 \pm 0.08$ ps^{-1}

B^0_s:

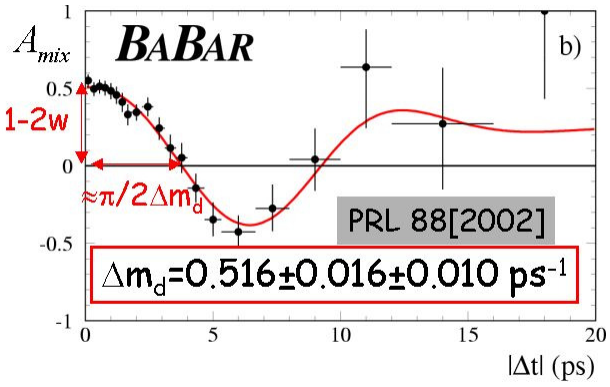
– $\Delta m_s > 16.6$ ps^{-1}



$\Delta m_s/\Delta m_d = V_{ts}/V_{td}$

Input to SM

$$A_{mix}(\Delta t) = \frac{N_{unmixed}(\Delta t) - N_{mixed}(\Delta t)}{N_{unmixed}(\Delta t) + N_{mixed}(\Delta t)} \approx (1-2\langle w \rangle)\cos(\Delta m \Delta t)$$



Bd mixing very well measured

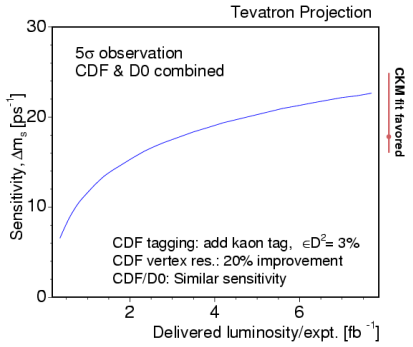
Bs mixing

Status now:

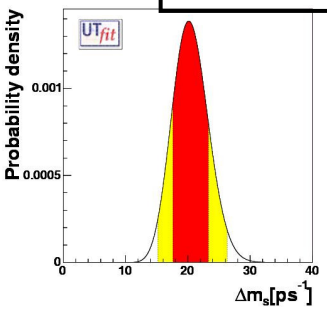
Unobserved: xs = Δm/Γ(>16.6 ps^{-1}@95%)

SM (UTfit) prediction of ~20ps^{-1}

Tevatron **may** observe mixing by 2007



Fit to experimental data



CP violation

CP violation:

CP violation **observed** in K^0-$\overline{K}^0$ system
$C_+ \neq C_-$ $\quad C_+/C_- = (1+\varepsilon)/(1-\varepsilon)$, $\varepsilon \sim 2.10^{-3}$

Must be included in SM

CKM matrix allows CP violation (but does not predict magnitude)

Nb. Big Bang – equal matter:antimatter. Universe now – matter.

Need quite a lot of CP violation for this to happen.

CP violation

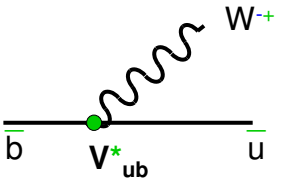
CP violation if b, anti-b decay rates different

Unitary CKM matrix ⇒ 4 free parameters (3 angles; **1 phase = CP**)

CKM **elements** containing **b quarks most sensitive** to phase

Quarks

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



Antiquarks

$$\begin{pmatrix} \overline{d}' \\ \overline{s}' \\ \overline{b}' \end{pmatrix} = \begin{pmatrix} V^*_{ud} & V^*_{us} & V^*_{ub} \\ V^*_{cd} & V^*_{cs} & V^*_{cb} \\ V^*_{td} & V^*_{ts} & V^*_{tb} \end{pmatrix} \begin{pmatrix} \overline{d} \\ \overline{s} \\ \overline{b} \end{pmatrix}$$

4 parameters

- If CKM parameterisation is correct
  - All mixing and weak decays must give consistent results
  - The phase you extract from CP violation in the K system will predict the amount of CP violation in the B system
- Make many measurements and test whether CKM formalism works
- Note: Doesn't exactly *explain* origin of CP

CP violation

- **Need** 3 generations of quarks to introduce CP violation into theory

$$\begin{pmatrix} d_W \\ s_W \\ b_W \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Mixing matrix is 3x3.

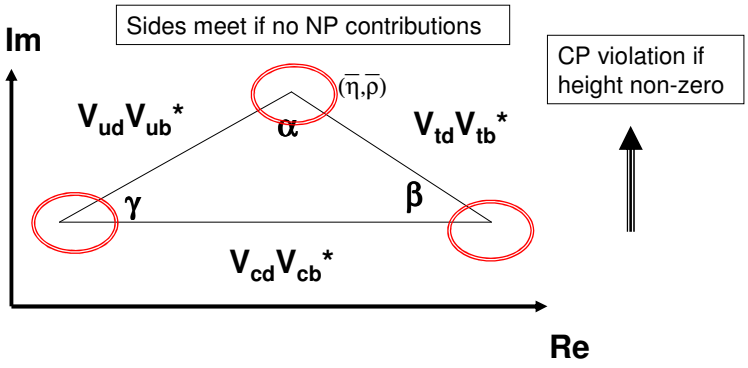
Unitarity constraints $\Rightarrow$ 4 independent parameters

3 angles quantify mixing between (1,3) (2,3) (1,2) generations,
1 complex phase (mechanism for introducing CP)

CP Angles

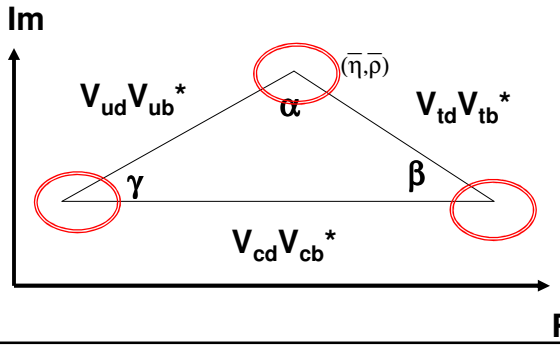
Unitarity relations often represented graphically ……..

$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$



CP Angles

Angles α,β,γ just combinations of CKM matrix elements …..



$$\alpha = \arg\left\{-\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}}\right\}$$

$$\beta = \pi - \arg\left\{\frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}}\right\}$$

$$\gamma = \arg\left\{-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right\}$$

CKM matrix

Also **Wolfenstein parameterisation** $O(\lambda^3)$:

To ensure unitarity

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

$$\lambda = s_{12} \quad A = \frac{s_{23}}{s_{12}^2} \quad \rho = \frac{s_{13}\cos\delta}{s_{12}s_{23}} \quad \eta = \frac{s_{13}\sin\delta}{s_{12}s_{23}}$$

$\sin\theta_c$

$$\lambda = \sin\theta_{12} \approx 0.22$$

Relating parametrisations

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

$-|V_{ub}|e^{-i\gamma}$

$-|V_{td}|e^{-i\beta}$

$|V_{ts}|e^{-i\delta\gamma}$

β = B^0_d mixing phase
δγ = B^0_s mixing phase
γ = weak decay phase

If η=0, no CP violation

$$\beta = \tan^{-1}\left[\frac{\bar{\eta}}{1-\bar{\rho}}\right]$$

$$\gamma = \tan^{-1}\frac{\eta}{\rho}$$

$$\delta\gamma = \eta\lambda^2$$

CP violation in B^0 sector

- CP violation in B^0 sector expected to be large (complex CKM matrix elements involving b are large)
- Experimentally –
  - Choose decay channel, measure asymmetry A_{CP}

measured

Δm measured in mixing

$$A_{CP} = \frac{\Gamma(B(t)\to f) - \Gamma(\bar{B}(t)\to f)}{\Gamma(B(t)\to f) + \Gamma(\bar{B}(t)\to f)} = \frac{A_f^{dir}\cos(\Delta m t) + A_f^{mix}\sin(\Delta m t)}{\cosh(\Delta\Gamma t/2) - A_f^{\Delta}\sinh(\Delta\Gamma t/2)}$$

$\lambda_f = (q/p)(\bar{A}_f/A_f)$

$A_f^{dir} = (|\lambda_f|^2-1)/(|\lambda_f|^2+1)$

$A_f^{mix} = 2\,\mathrm{Im}(\lambda_f)/(|\lambda_f|^2+1)$

$A_f^{\Delta} = 2\,\mathrm{Re}(\lambda_f)/(|\lambda_f|^2+1)$

Ratio of CKM matrix elements

$\Delta\Gamma = 0$ for B^0, $\Delta\Gamma/\Gamma \sim 0.1$ for B_s

CP violation in B^0 sector

- Eg. $B^0 \to \pi^+ \pi^-$ asymmetry

$$A_{CP} = \frac{\Gamma(B(t)\to f) - \Gamma(\bar{B}(t)\to f)}{\Gamma(B(t)\to f) + \Gamma(\bar{B}(t)\to f)} = \sin(\Delta m t)\ \mathrm{Im}\left[\frac{q}{p}\frac{\bar{A}f}{Af}\right]$$

measured

Measured in mixing

Ratio of CKM matrix elements

$\sin 2\alpha$

p, q original admixtures of B^0, $\bar{B}^0$ in mass eigenstates (ratio of CKM matrix elements) – like C_+, C_-

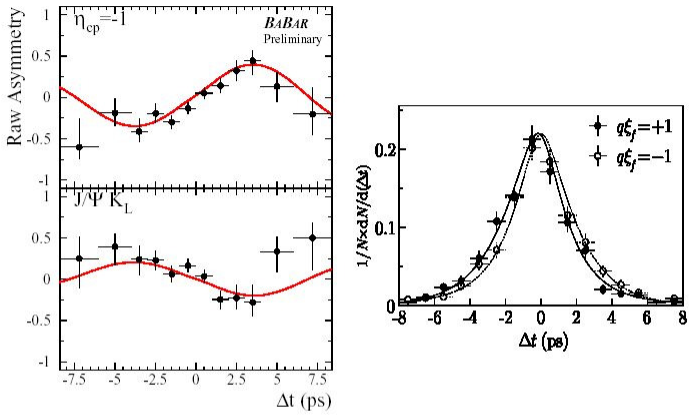


Figure 1: *On the left, raw asymmetry in the number of B^0 and $\overline{B}^0$ tags, $(N_B^0 - N_{\overline{B}^0})/(N_B^0 + N_{\overline{B}^0})$, as a function of Δt from Babar. The upper plot is for CP odd final states, the lower plot for the CP even $J/\psi K_L^0$. The solid curves represent the result of the combined fit which has determined the central* $\sin 2\beta$ *value (0.75). On the right, Δt distributions from Belle for the events with $q\xi_f = 1$ and $q\xi_f = -1$, where ξ_f is the CP eigenvalue and $q = 1(-1)$ for B^0 ($\overline{B}^0$) tag. The result of the global fit for* $\sin 2\beta = 0.82$ *is shown as solid and dashed curves respectively.*

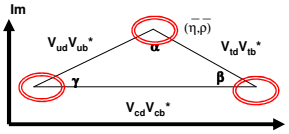
CP violation in B^0 sector

Other decays involve other CKM matrix couplings, hence other "angles" β, γ

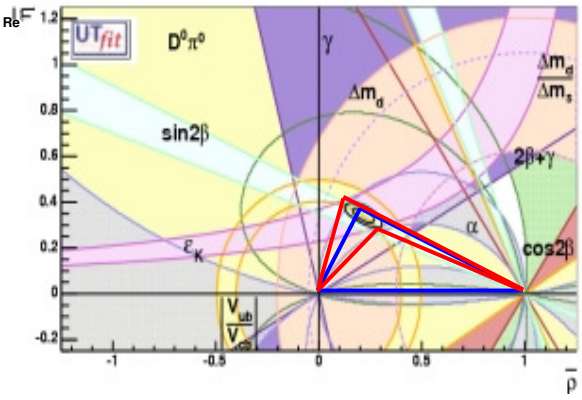
- $B^0_d \rightarrow J/\Psi\ K^0_s$; A_{CP} α sin 2β
- $B^0_d \rightarrow D^0 K^{*0}$; A_{CP} α sin γ

- Huge program of work at Babar, Belle (B^0_d), CDF, D0 (B^0_s) to investigate consistency of SM relationships
  - Future work planned at LHCb

CP violation studies



Current status

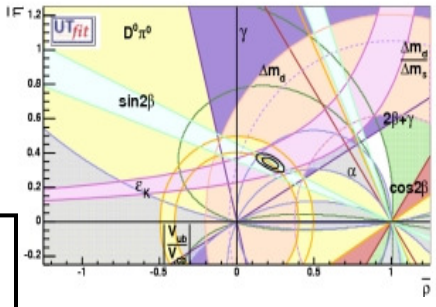


β,measured well at Babar, Belle

α can be measured well at Babar, Belle

γ can only be measured well at LHCb

But



Although CP violation measurements in SM consistent

Can only explain conversion of ~1 galaxy worth of matter in universe!

There must be another CP violating mechanism (which we don't know about in SM)

Review

- Weak, strong quark eigenstates are different
- Admixtures quantified in CKM matrix
  - Matrix can be parametrised as 3 angles, 1 phase
  - Phase is mechanism in theory for CP violation
  - No SM predictions for matrix parameters
  - Measurements underway to test SM predictions of relationships