Lecture 4

- Recap of PHYS110 *- 1 lecture*
- **Physical Optics** *- 4 lectures*
  - EM spectrum and colour
  - Light sources
  - **Interference and diffraction**
  - Polarization
- Lens Aberrations *- 3 lectures*
  - Spherical aberrations
  - Coma, astigmatism, field curvature, distortion
  - Chromatic aberrations
- Instrumental Optics *- 4 lectures*
  - Telescope, microscope
  - Stops, eyepieces
  - Instruments for the anterior eye
  - Instruments for the posterior eye

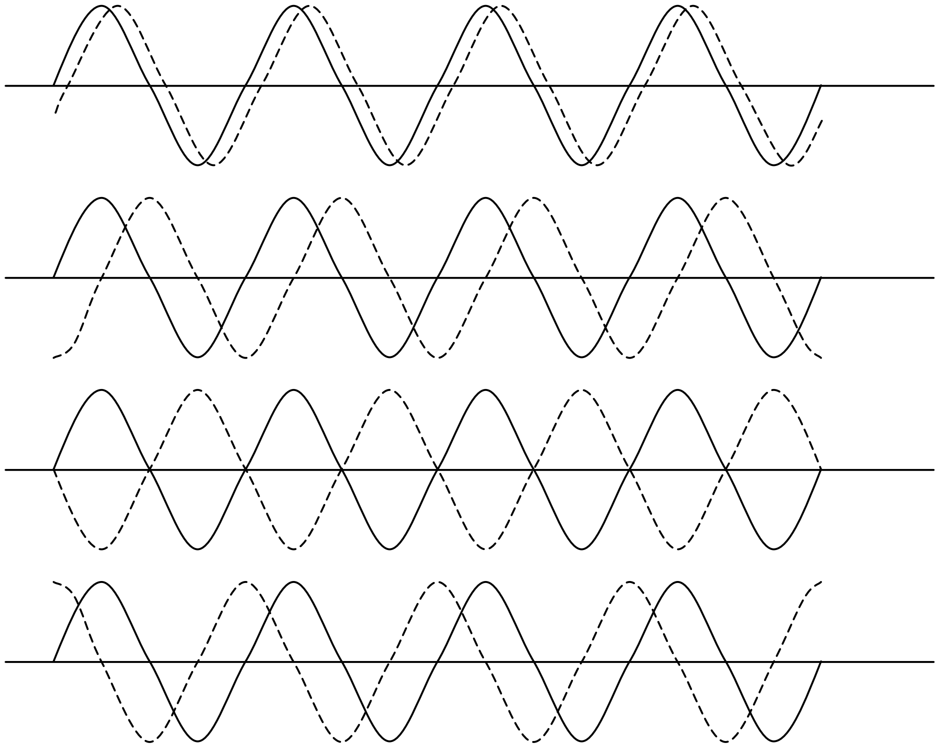
Lecture 4

- Interference and diffraction
  - Interference
  - Anti-reflective coatings
  - Single- and double slit diffraction
  - Diffraction gratings
  - Airy's disk and Raleigh's criterion

Phase difference

- Two waves of the same wavelength can differ in **phase**

phase difference



$1/12\ \lambda = 30^{o}$

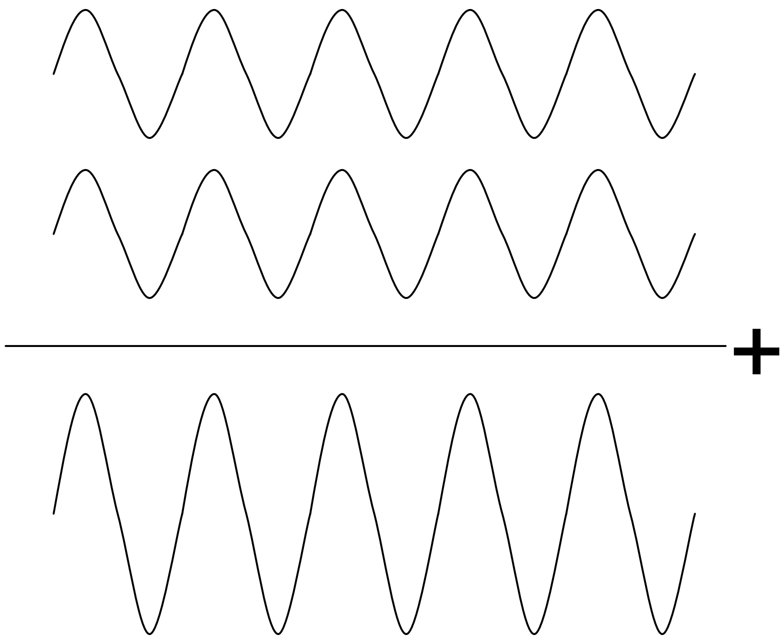
$1/4\ \lambda = 90^{o}$

$1/2\ \lambda = 180^{o}$

$3/4\ \lambda = 270^{o} = -90^{o}$

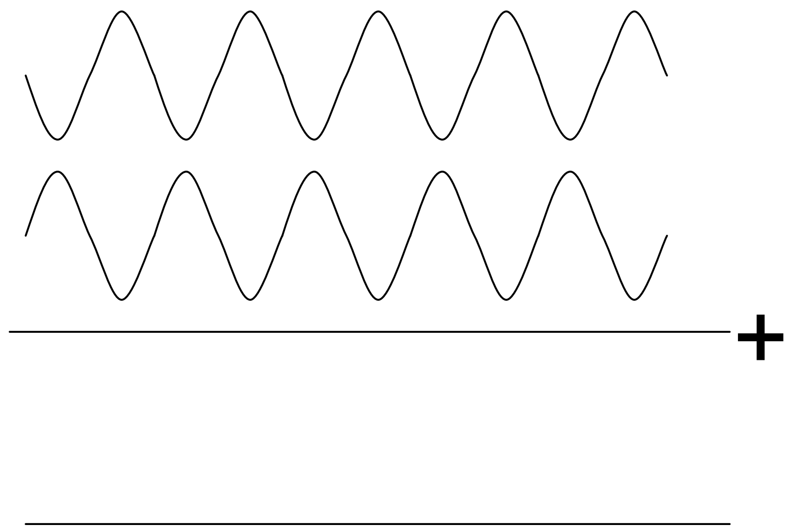
Interference

"in-phase"
phase difference = 0
or an integer number
of wavelengths 1,2,3,...



Constructive interference

"out-of-phase"
phase difference =
half a wavelength
or a half-integer number
of wavelengths 1½, 2½, 3½,...

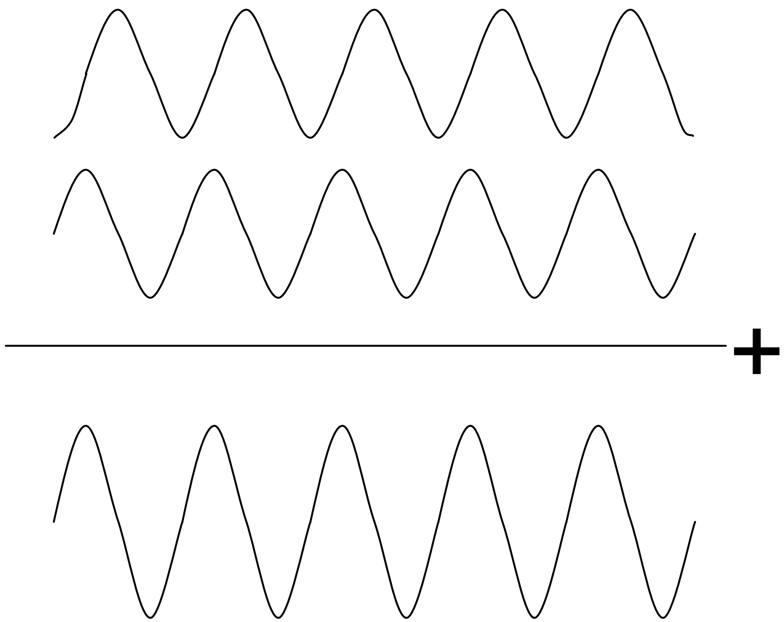


Destructive interference

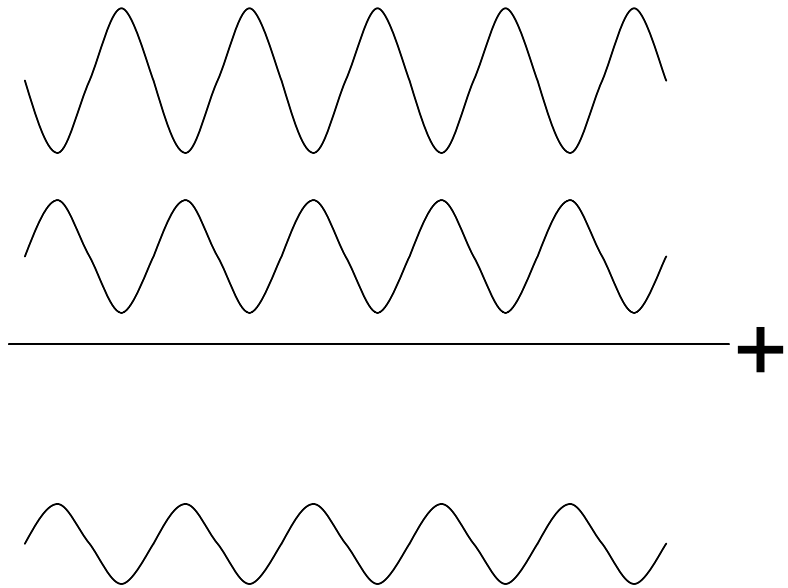
Partial interference

- Less extreme interference is also possible

phase difference =
quarter wavelength



"out-of-phase"
but different amplitude



Two waves that have different amplitudes cannot fully cancel each other

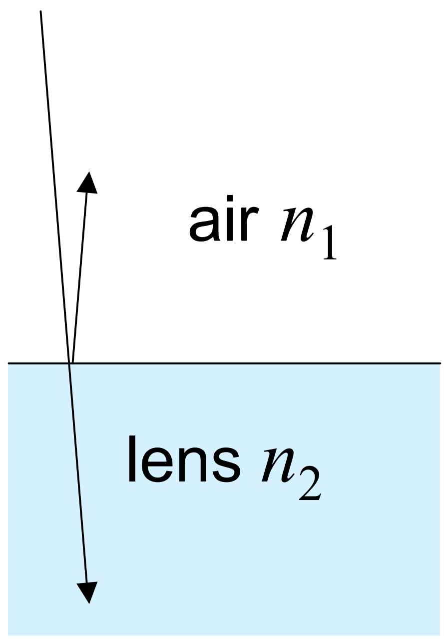
Anti-reflective coating

- In going from one medium to the other (for example air to glass) some fraction of the light will be reflected

- For perpendicular incidence, the fraction R of reflected light is

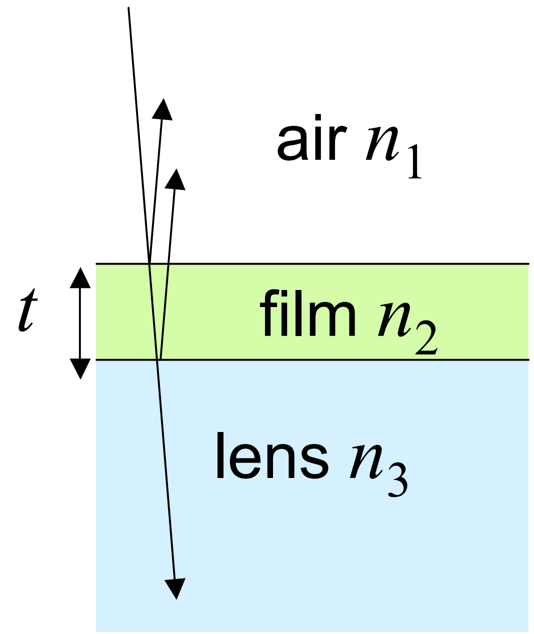
$$R = \left(\frac{n_2 - n_1}{n_2 + n_1} \right)^2$$

- This reflection can be reduced by applying a thin coating



Anti-reflective coating

- By applying a thin coating on the lens, the light is reflected twice
- If the optical pathlength between the two reflections is a half-integer number of wavelengths, the two reflections interfere destructively and the reflection is reduced
- This corresponds to a film thickness of ¼, ¾, 1¼, ... wavelength
- ¼ wavelengths is common: **quarter-wavelength coating**



Anti-reflective coating

- Recall, in a medium with refractive index $n>1$, the wavelength is shorter than in air.
- The quarter-wavelength coating should be the thickness of a quarter wavelength ***inside the coating***
- The right thickness is thus:

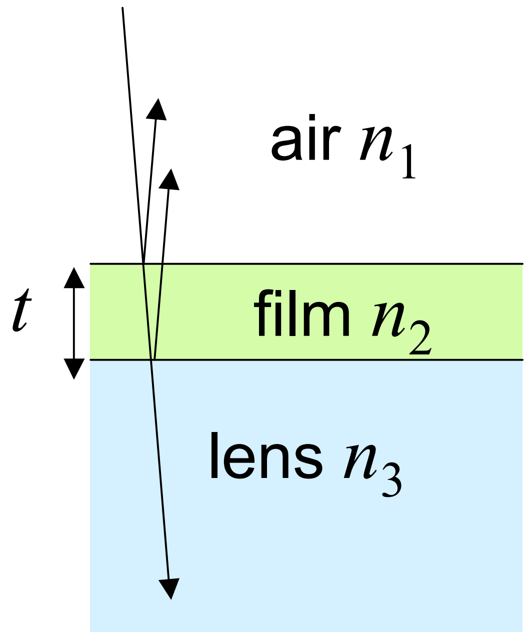
$$t = \frac{1}{4}\frac{\lambda}{n_2}$$

Anti-reflective coating

- For fully destructive interference the amplitudes of the two reflections should be equal
- That is achieved by choosing

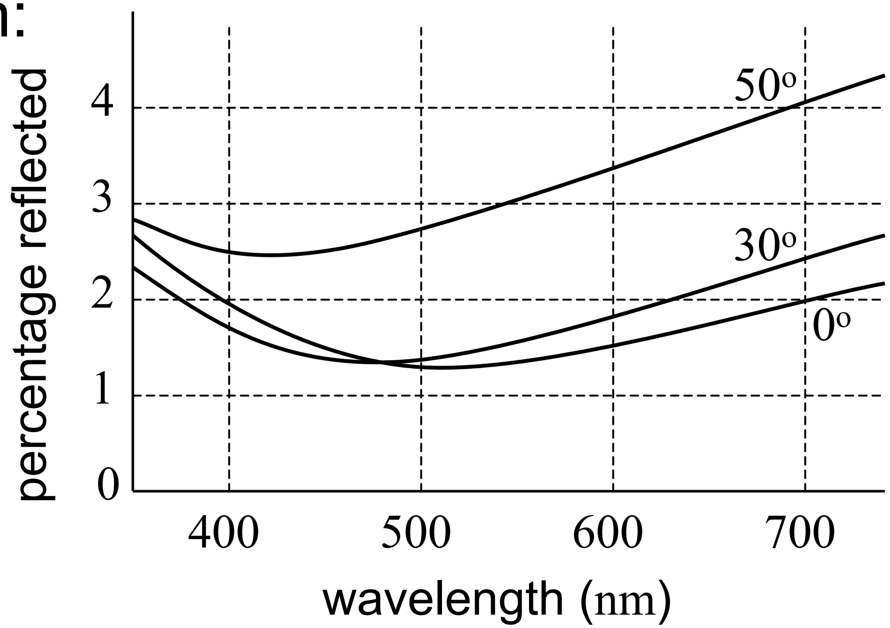
$$n_2 = \sqrt{n_1 \cdot n_3}$$

- For glass with $n_3 \approx 1.5$ this means n_2=1.22
- In practice no material with such low refractive index exists and magnesium-fluoride (n=1.38) is often used
- Zero reflection is not possible with such coating



Anti-reflective coating

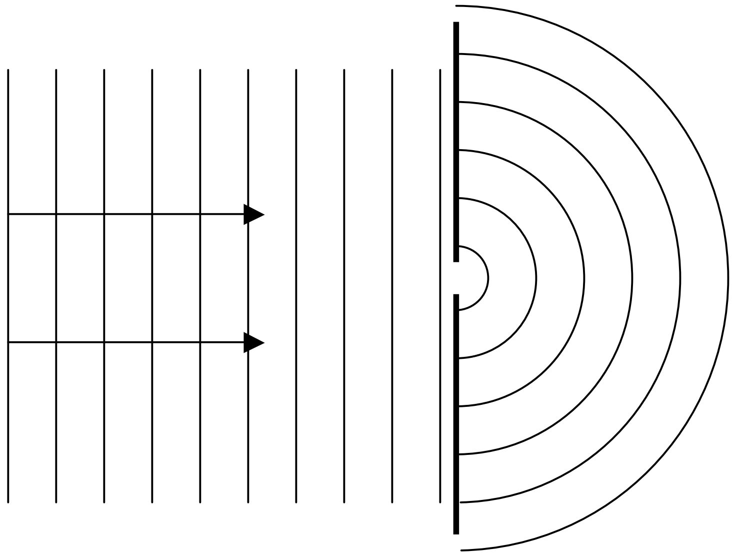
- The thickness of anti-reflective coating is optimal for one wavelength and one angle of incidence only
- By choosing optimal suppression of reflection for green light (550nm), reasonable suppression is achieved over the whole visible spectrum:



- Further reduction of reflections can be achieved by applying multiple layers of coating

Single slit diffraction

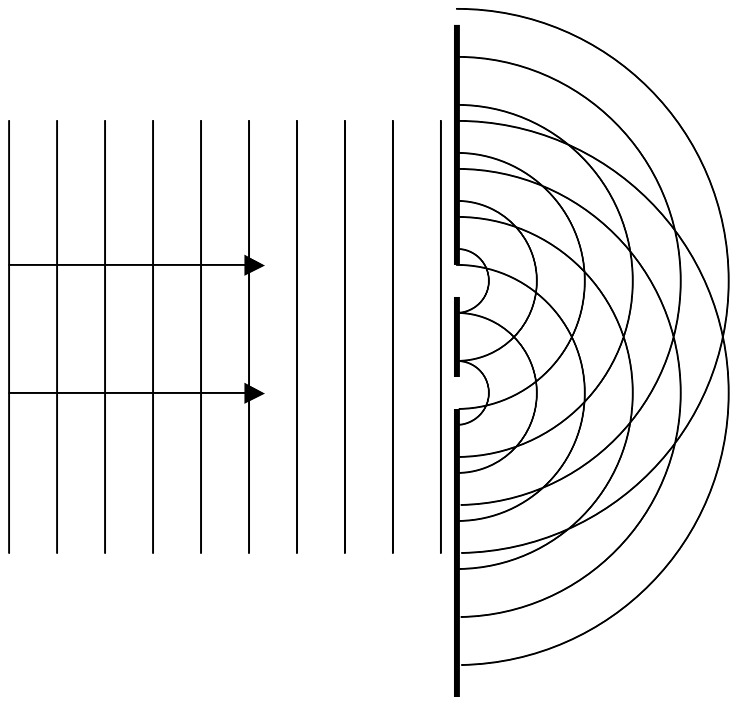
- If a parallel beam of light (e.g. laser) falls on a very narrow slit (<λ), secondary waves are generated as if there was a source of light at the slit



- The spreading of light at the narrow opening is called ***diffraction***

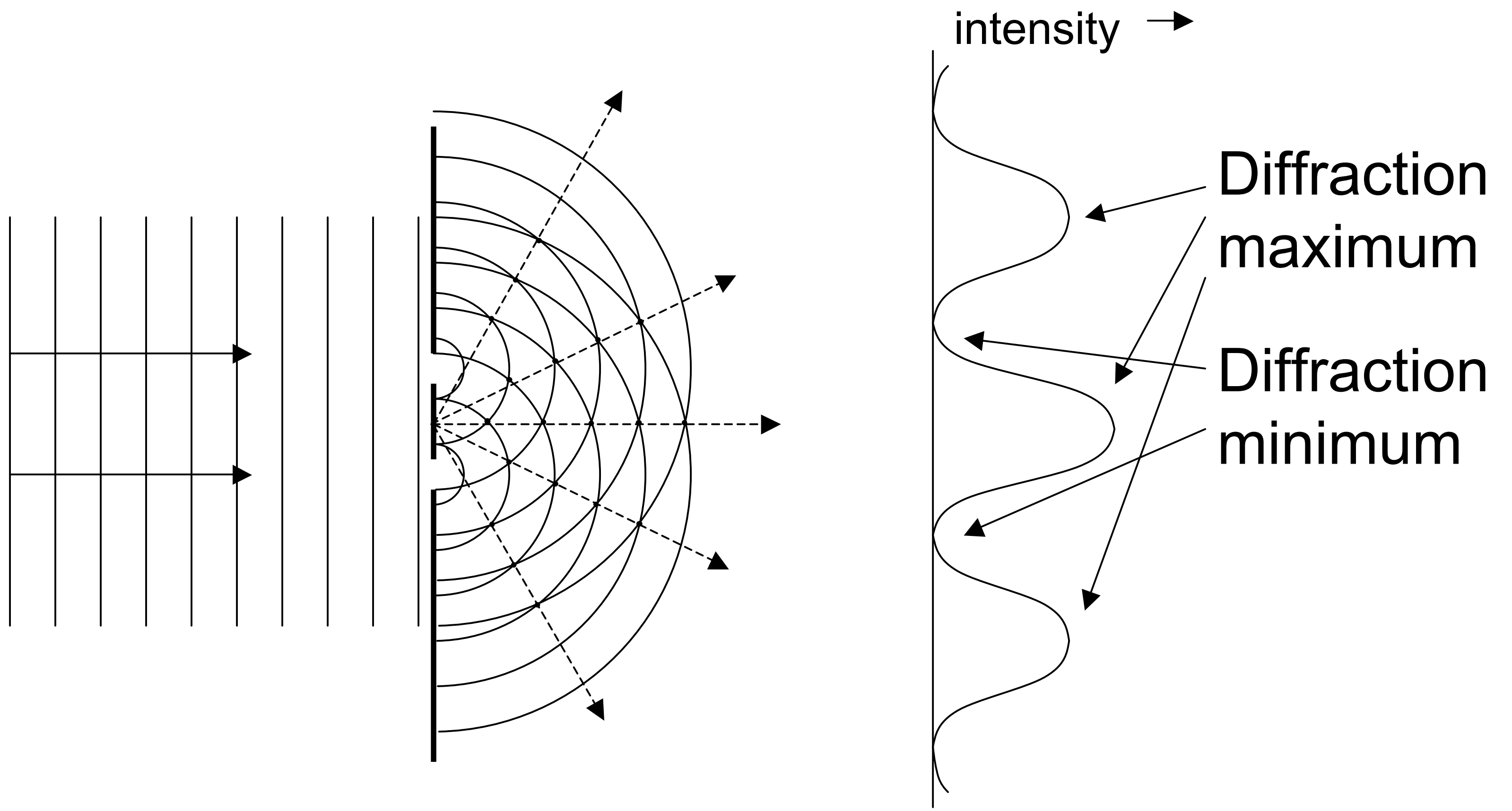
Double slit diffraction

- When the laser strikes two slits that are close together, two sets of waves will be generated that will interfere in the region beyond the slits



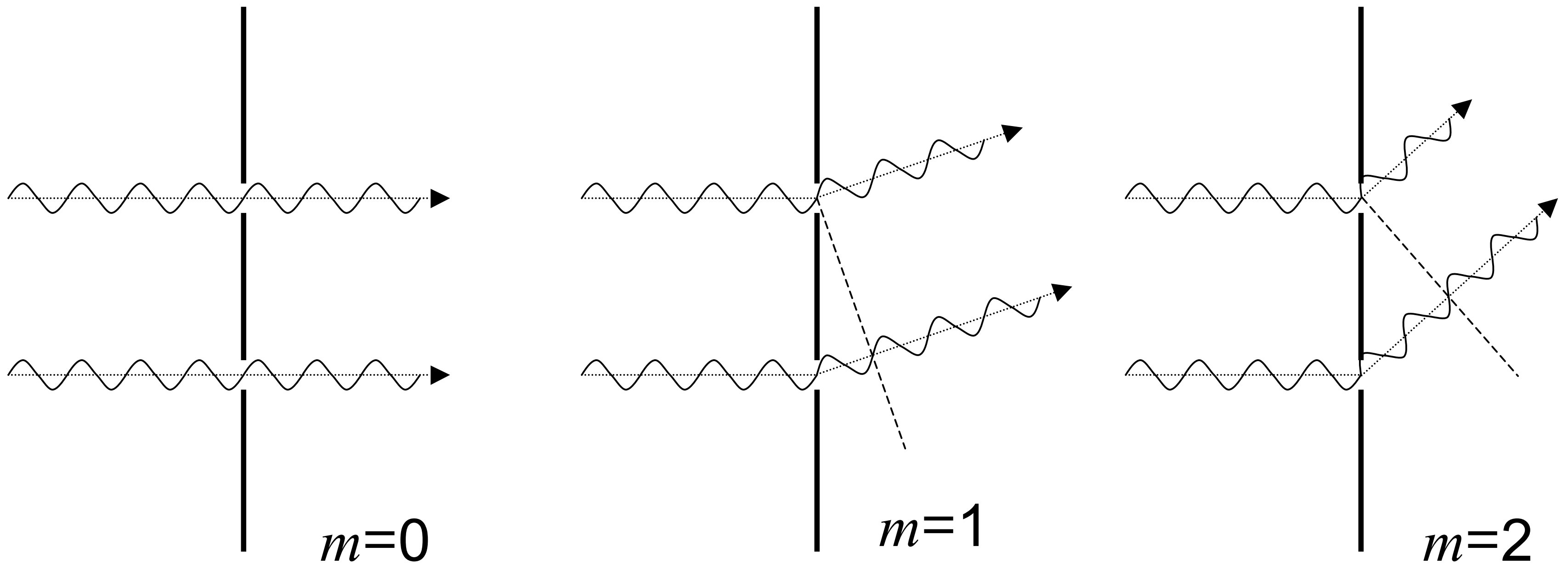
Double slit diffraction

- In the directions where the wavefronts overlap one has constructive interference (diffraction maxima)



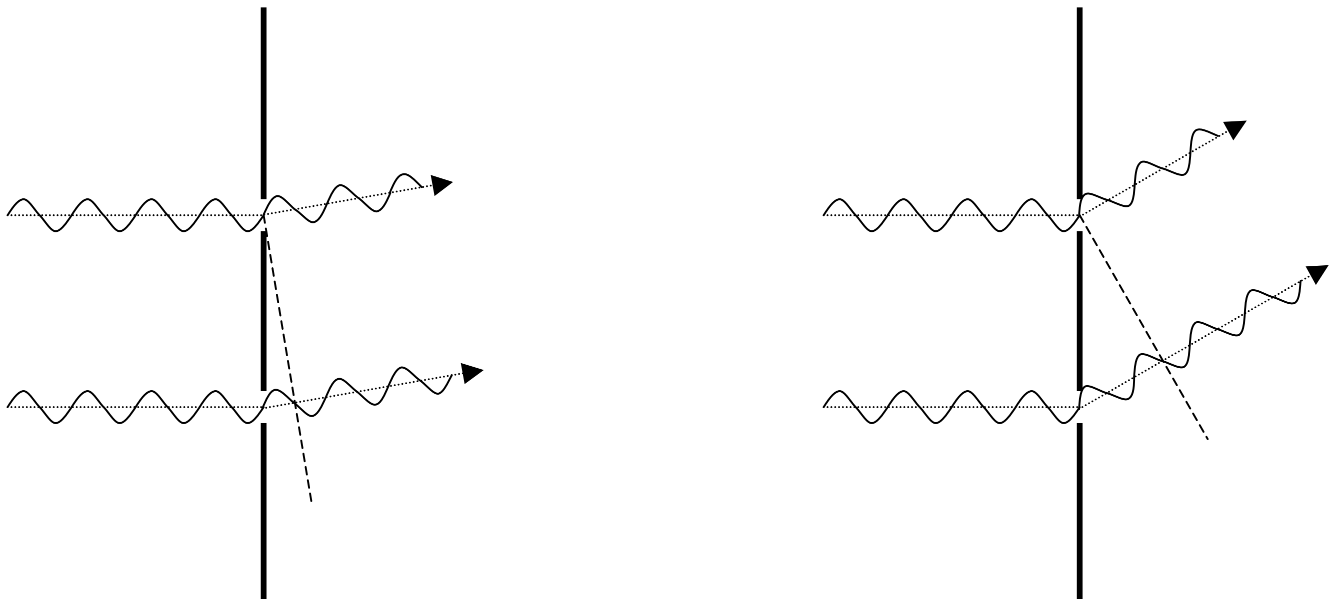
Double slit diffraction

- If the pathlength between the light coming through the two slits is an **integer** number m (0,1,2,3, ...) wavelengths, **constructive** interference occurs:



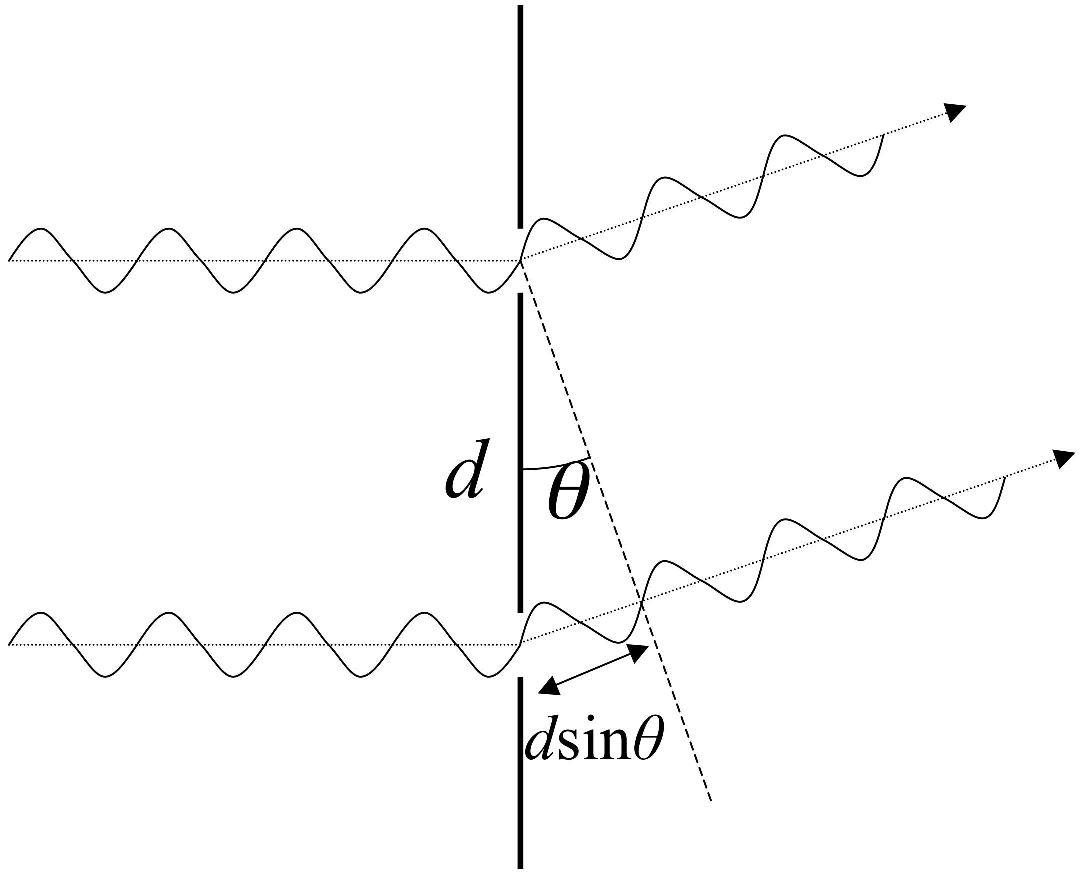
Double slit diffraction

- If the pathlength between the light coming through the two slits is a **half integer** number (½,1½,2½, ...) of wavelengths, **destructive** interference occurs:



Double slit diffraction

- The pathlength difference is the distance d between the two slits times the sine of the outgoing angle θ : $d \sin\theta$



Double slit diffraction

- Constructive interference (diffractive maxima) occur at angles θ to the beam direction given by:

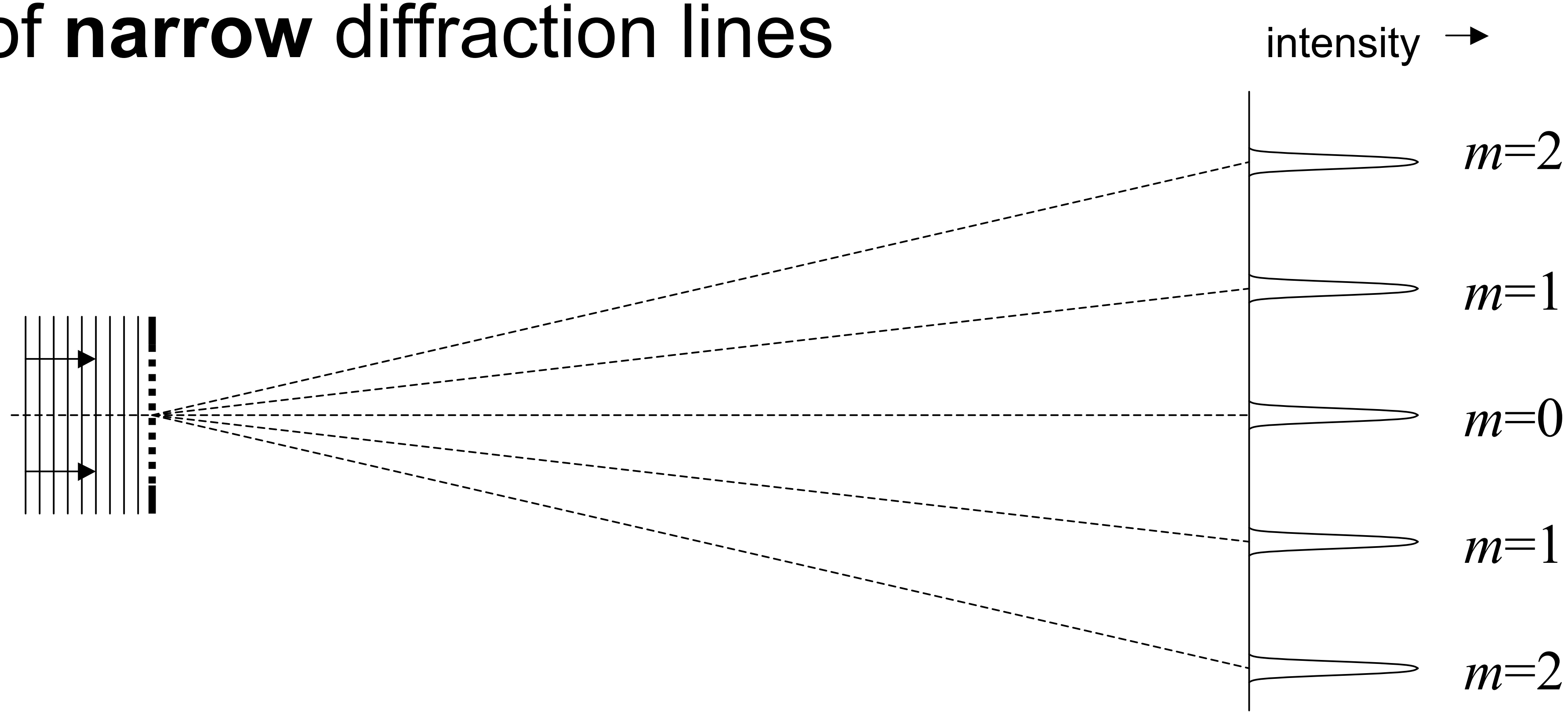
  $d \sin\theta = m\lambda$, where $m = 0,1,2,3,\ldots$

- Destructive interference (diffractive minima) occur at angles θ to the beam direction given by:

  $d \sin\theta = (m+½)\lambda$, where $m = 0,1,2,3,\ldots$

Diffraction grating

- A diffraction grating is a series of **many** narrow closely spaced slits
- Monochromatic light is diffracted into a series of **narrow** diffraction lines

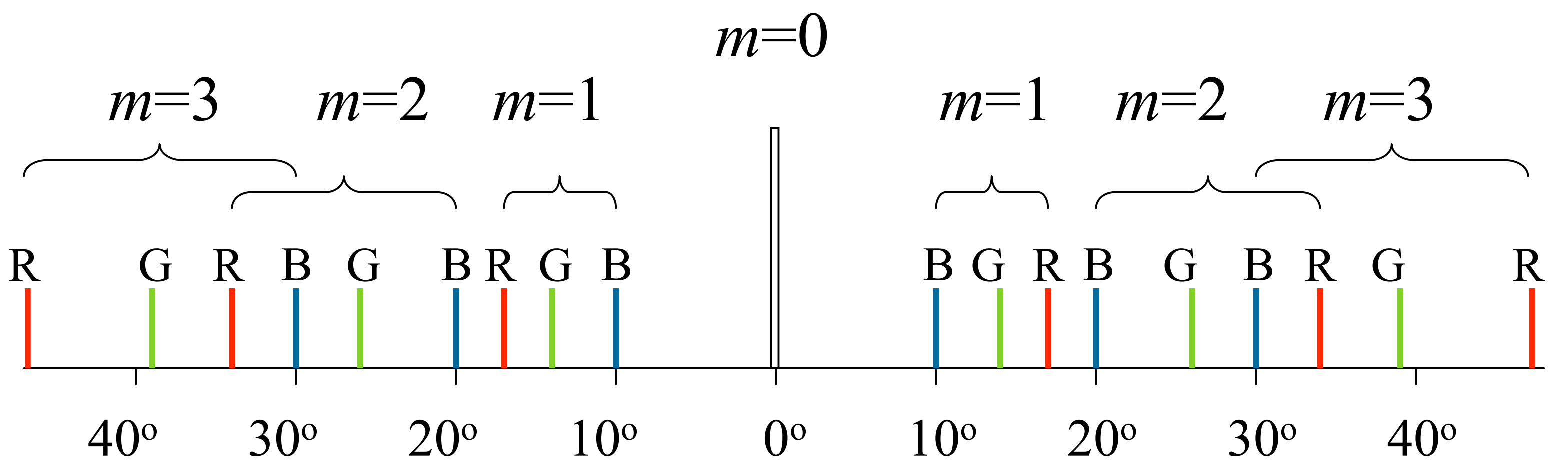


Diffraction grating

- Just as for the two-slit diffraction, the distance between the maxima is given by $d\sin\theta = m\lambda$
- Because of the narrow maxima, a diffraction grating can be used to measure the wavelength of a light source

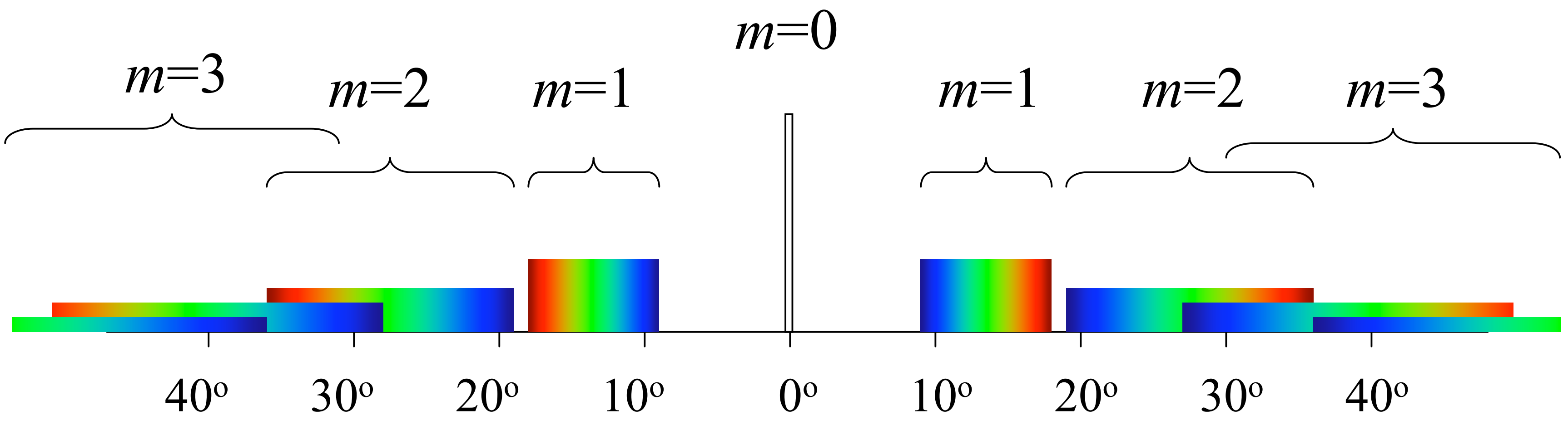
Spectral analysis

- A diffraction grating can be used to analyse the spectrum of a light source
- For example a gaseous light source with 3 discrete spectral lines gives the following diffraction pattern:



Spectral analysis

- A solid light source with a continuous spectrum will show its complete spectrum when analyzed with a grating:

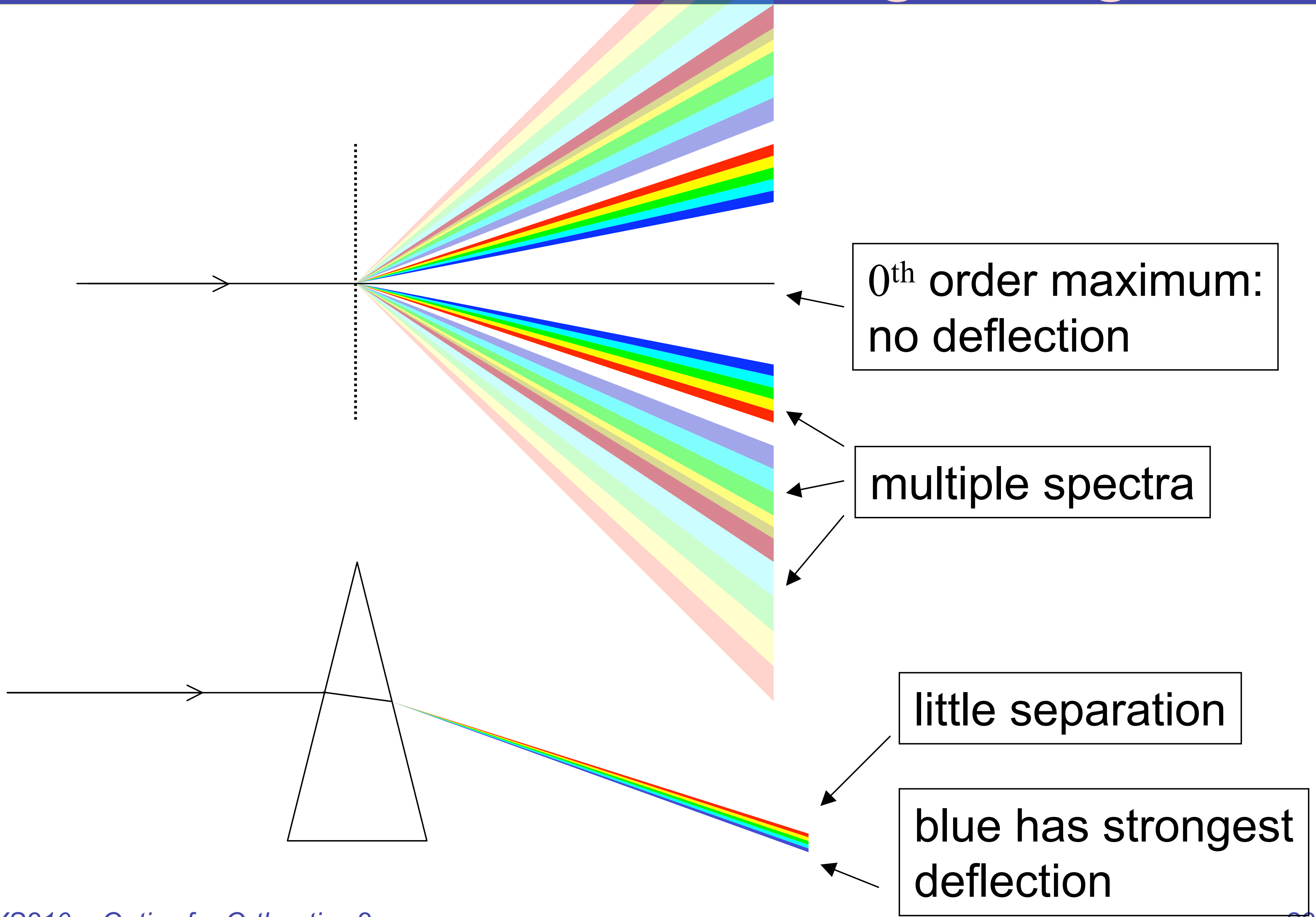


- Only the first order spectrum is clear, the 2nd, 3rd, etc. overlap

Prism vs diffraction grating

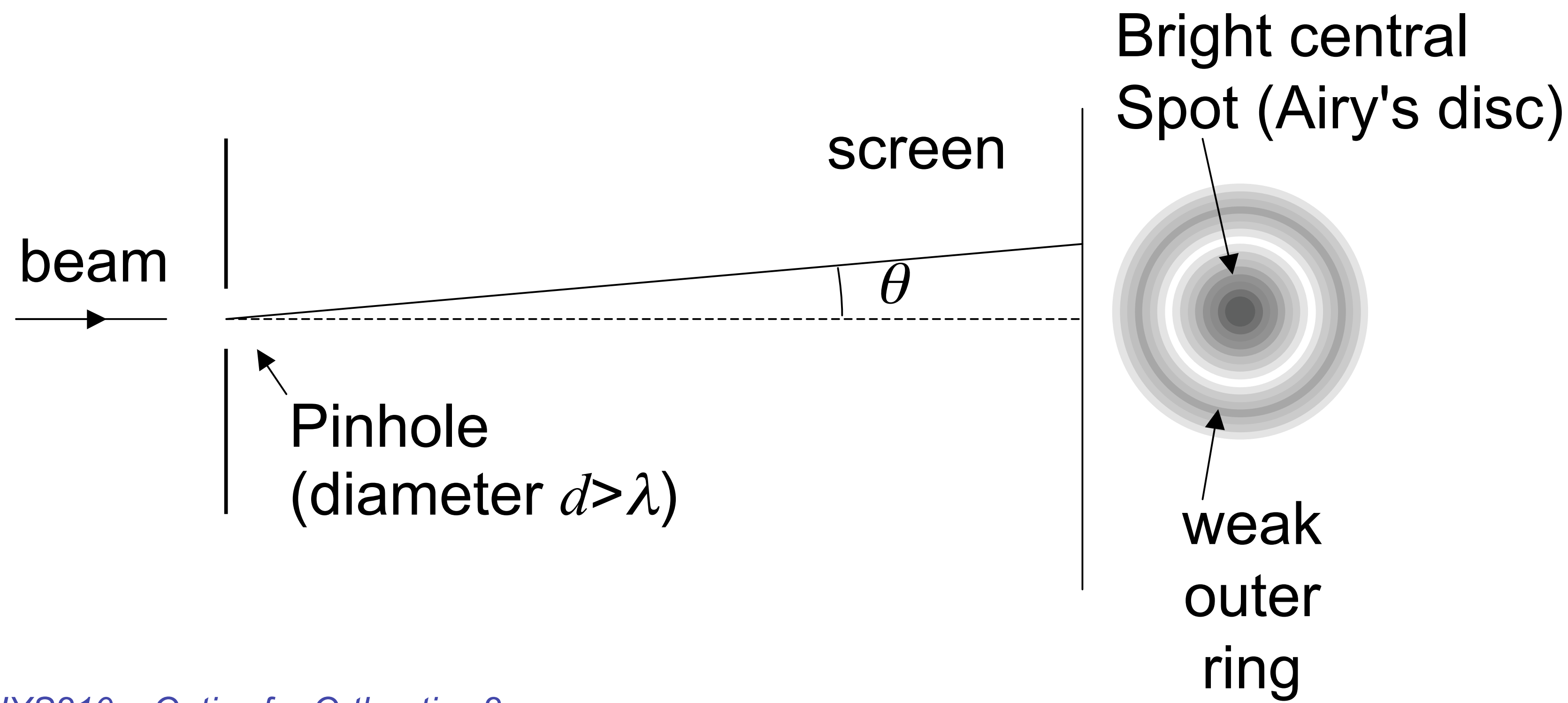
- We have now seen two different ways to do a spectral analysis of light: a prism through dispersion and a grating through diffraction. There are some differences:
  - A diffraction grating has a 0^{th} order maximum with light that passes straight through, a prism deflects all light.
  - A diffraction grating splits the light into multiple spectra, a prism generates just one spectrum
  - A diffraction grating separates the colours much stronger than a prism does
  - A prism deflects short wavelengths (blue) more than long wavelengths (red). A diffraction grating deflects red more than blue

Prism vs diffraction grating



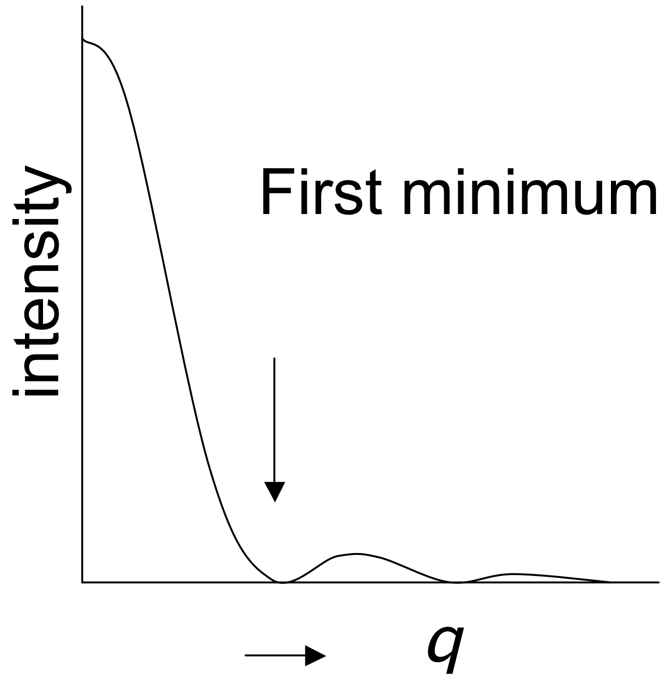
Diffraction through circular opening

- A beam of light striking a small pinhole produces a circular diffraction pattern on a screen beyond.
- The bright central spot is called **Airy's disc**.



Size of Airy's disc

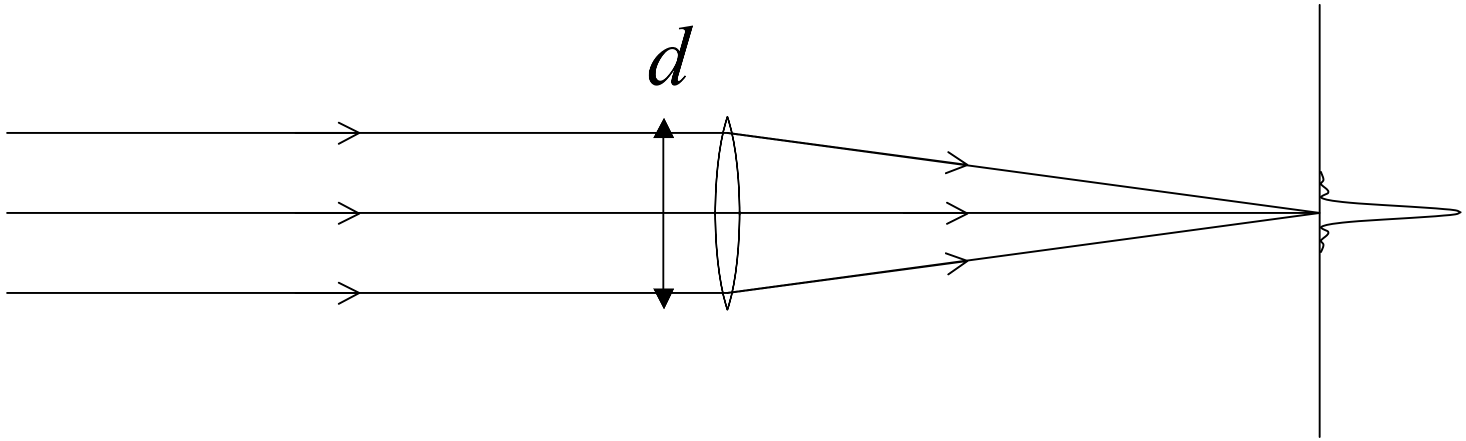
- The first minimum of Airy's disc occurs at
  $\sin\theta = 1.22\,\lambda/d$



- This angle is called the angular radius of Airy's disc

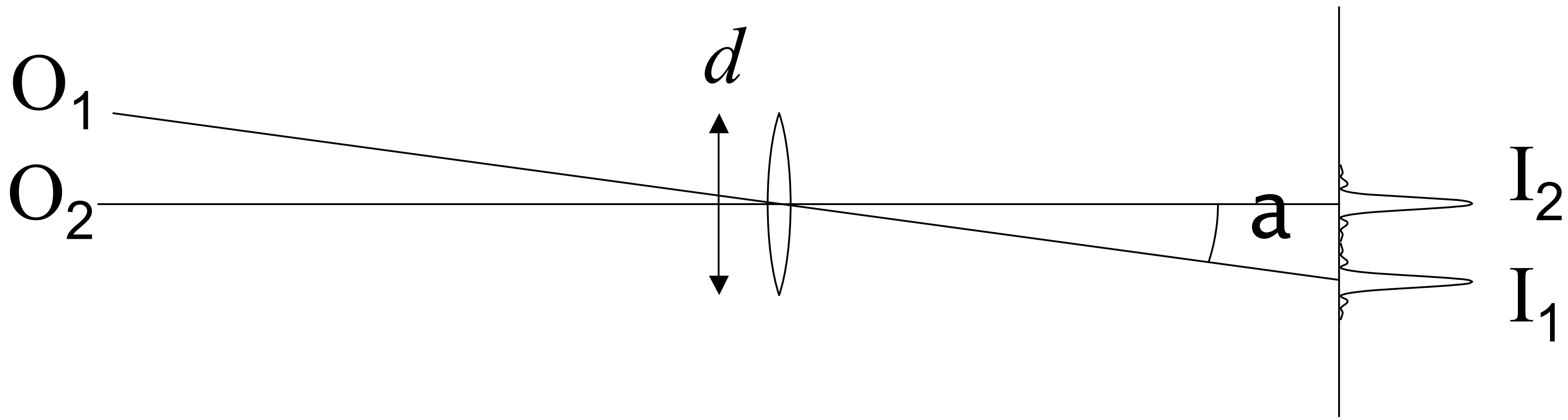
Airy's disc for an optical system

- The same holds for an optical system
- Parallel light impending on a perfect lens will not focus in a spot of zero size
- The diffraction limit of the spot size is Airy's disc with an angular size of $\sin\theta = 1.22\ \lambda/d$



Resolving power of optical system

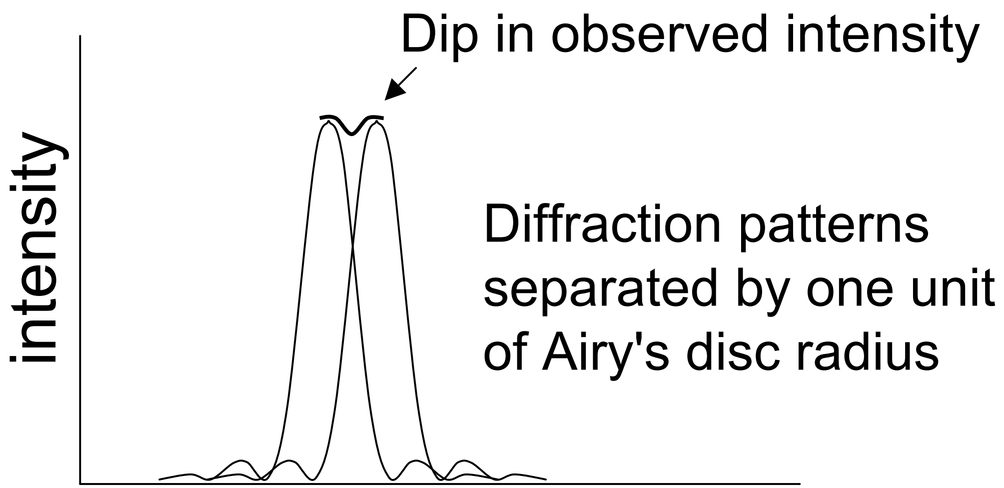
- Suppose there are two distant light sources O_1, O_2 (e.g. stars) separated by an angle *a* that produce two images I_1, I_2 in a telescope with a lens of diameter d
- The images will be two Airy discs with a radius given by $\sin\theta = 1.22\,\lambda/d$



- What is the smallest angle *a* that can be resolved?

Rayleigh's criterion

- Lord Rayleigh proposed the criterion that two points are just resolvable when the angular separation *a* equals the angular radius of Airy's disc
- At this radius there is a distinct dip in the intensity of the observed image between the two points



Resolution power

- We thus find the following relation for the resolution power of an optical system:

  $\sin a = 1.22\,\lambda/d$

- For small angles we have $\sin a \approx a$ and the relation simplifies to $a = 1.22\,\lambda/d$

Visual acuity of the eye

- For the eye, the resolution power is called the **visual acuity**
- Assuming a pupil diameter of $4\ \mathrm{mm}$ and a wavelength of $550\ \mathrm{nm}$ we find

$$\alpha = 1.22\frac{\lambda}{d} = 1.22\frac{550\cdot 10^{-9}\,\mathrm{m}}{4\cdot 10^{-3}\,\mathrm{m}} =$$

$$1.7\cdot 10^{-4} = 0.17\mathrm{mrad}$$

- This corresponds to being able to distinguish two points separated by $0.17\mathrm{mm}$ at a distance of $1\mathrm{m}$