Some Basics

Kinematics
Cross Section
Lifetimes
Spectroscopies

Particle Kinematics E. Byckling and K. Kajantie Wiley&Sons 1973

Introduction to Particle Physics - Max Klein – Lecture 2 - Liverpool University 25.2.13

Organisation

1. Four tutorials in week 6,8,10,11 Thursday afternoons, held by Carl Gwilliam, Barry King, Jan Kretzschmar, Steven Maxfield.

2. All lecture slides will be posted. There are no hand outs, please take notes.

3. Exam in May: 1.5 hours. Please consult Martin&Shaw, and also join the tutorials with solved exercises. The course has 7 double lectures, probably the last two will be devoted to summaries and exercises.

Tutorial for Thursday, 7^{th} of March

1. How large is the Thompson scattering cross section in barn?
2. Hofstatter used an electron beam of E_e=188 MeV, how large is β_e = velocity/c ?
3. In an experiment a particle decay at rest is observed into a muon and a neutrino. The mass of the muon is known to be M_μ=106 MeV and the kinetic energy of the muon is measured to be T=4.4 MeV. Determine the mass of the parent particle and identify it with a known particle.
4. The lifetime of the muon is given as $\tau_\mu = 192\pi^3/G_F^2M_\mu^5$. ($M_\mu$=0.11GeV, G=1.17 10^{-5} GeV^{-2}). Calculate τ_μ in seconds. How large is the tau lifetime (M_τ=1.78 GeV)?
5. The LHC operated at E_p= 3.5 TeV proton beam energy. How large is the cms energy in a pp collision? In Drell Yan Scattering at the LHC, a quark and an anti-quark of momentum fractions x_1 and x_2 interact to form a new state Y. What is the electric charge of Y? What is the mass of Y as functions of E_p, x_1 and x_2?
6. How was the muon neutrino discovered?

Hand in please latest by Wednesday, 6^{th} of March

Units

Convention in particle physics:

h = one unit of action (ML^2/T), c = one unit of velocity (L/T)

mass (m), momentum (mc), energy (mc^2) are of dimension [GeV]

length (h/mc), time (h/mc^2) are of dimension [GeV^{-1}]

Conventional Mass, Length, Time Units, and Positron Charge in Terms of $\hbar = c = 1$ Energy Units

Conversion Factor	$\hbar = c = 1$ Units	Actual Dimension
$1\ \text{kg} = 5.61 \times 10^{26}$ GeV	GeV	$\frac{\text{GeV}}{c^2}$
$1\ \text{m} = 5.07 \times 10^{15}\ \text{GeV}^{-1}$	GeV^{-1}	$\frac{\hbar c}{\text{GeV}}$
$1\ \text{sec} = 1.52 \times 10^{24}\ \text{GeV}^{-1}$	GeV^{-1}	$\frac{\hbar}{\text{GeV}}$
$e = \sqrt{4\pi\alpha}$	—	$(\hbar c)^{1/2}$

Units

Thompson scattering
eγ → eγ cross section:

$$\sigma = \frac{2}{3}\alpha^2 4\pi R_e^2$$

$$R_e = \frac{\hbar}{m_e c}$$

$$\alpha = 4\pi e^2 \cong \frac{1}{137}$$

$$\sigma = \frac{8\pi}{3}\alpha^2 \frac{\hbar^2}{m_e^2 c^2}$$

$$\sigma = \frac{8\pi}{3}\left(\frac{\alpha}{m_e}\right)^2$$

$$[\sigma] = m^2$$

$$\sigma = \frac{8\pi}{3}\left(\frac{\alpha}{m_e}\right)^2 \cdot \hbar^a c^b$$

$$a = +2$$

$$b = -2$$

$$\sigma = \frac{8\pi}{3}\alpha^2 \frac{(\hbar c)^2}{(m_e c^2)^2}$$

$$m_e c^2 = 0.51 \cdot 10^{-3} GeV$$

$$\hbar c = 2 \cdot 10^{-16} m \cdot GeV$$

← 1 GeV beam probes 0.2fm

$$GeV^{-2} = 0.4mb, 1b = 10^{-28} m^2$$

Formulae are usually written in ‘natural units’. However, in practical calculations, decay times are in seconds or cross sections in m^2. A procedure is to reintroduce powers of h and c which can be determined from dimensional counting.

Scattering Kinematics

a+b → 1+2+..+n
exclusive (e.g.: pp→pp elastic)
a+b→1+X
inclusive (ep→eX deep inelastic)
a+b→ 1+2+3+X
semi-inclusive (πp→ππpX)

$E_a+E_b=\Sigma E_i$
energy conservation
E=M+T
rest energy + kinetic
(c=1)
$(p_a+p_b)^2=(\Sigma p_i)^2$
4 momentum conservation
$p_i^2=E_i^2-\Sigma(k_i)^2=m_i^2$

if (neutrino, e) m is small:
energy=|3 momentum|

Conservation laws determine
phase space of reaction
(3n-4) variables in 2 → n scattering

Decay

c→ 1+2+…+m

$E_c=\Sigma E_j$

at rest: $\mathbf{k}_c=0$: $E_c=m_c$

"**crossing**" relates decay to scattering:
$p_a=p_c$, $p_b=-p_m$

Examples for Decays and the Conservation of Quantum Numbers

$\mu^- \rightarrow \nu_\mu e^- \overline{\nu}_e$	100%
$\tau^- \rightarrow \nu_\tau e^- \overline{\nu}_e$	17%, **branching ratio**
$n \rightarrow p e^- \overline{\nu}_e$	β decay

Four-vectors (an example)

Decay at rest: A → B + e
Calculate the electron energy
as a function of the masses of A,B and e

$$\vec{p}_A = 0$$

$$\vec{p}_B = -\vec{p}_e$$

$$E_e^2 = M_e^2 + \vec{p}_e^2$$

$$E_e^2 = M_e^2 + \vec{p}_B^2 = M_e^2 - M_B^2 + E_B^2$$

$$E_B^2 = (E_A - E_e)^2 = (M_A - E_e)^2$$

$$E_e^2 = M_e^2 - M_B^2 + M_A^2 - 2M_A E_e + E_e^2$$

$$E_e = \frac{M_A^2 - M_B^2 + M_e^2}{2M_A}$$

The energy of the decay electron is exactly determined by the kinematics of the decay.

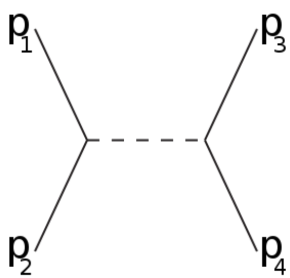
In the β decay, however, a continuous energy distribution, was measured.

This was one of the major puzzles in particle physics in the early thirties

-Bohr: no energy conservation
-Heisenberg: space-time modifications
-Pauli: n → p e + neutron(neutrino)

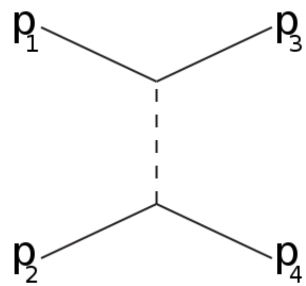
Mandelstam Variables

s-channel



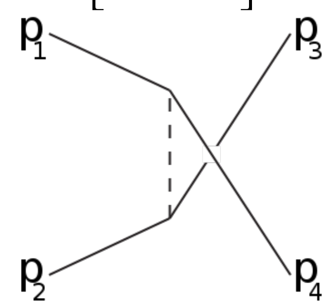
s= cms energy squared

t-channel



t= 4-momentum transfer squared

u-channel [crossed]



Feynman Diagrams

$s=(p_1+p_2)^2=(p_3+p_4)^2$ $\quad$ $t=(p_1-p_3)^2=(p_2-p_4)^2$ $\quad$ $u=(p_1-p_4)^2=(p_2-p_3)^2$

$p^2=M^2$ if the masses are negligible, these variables become 4-vector products:

$s=2p_1p_2=2p_3p_4$ $\quad$ $t=-2p_1p_3=-2p_2p_4$ $\quad$ $u=?$

Calculate the sum $s+t+u=?$

Frames of Reference – The Quest for Colliders

Laboratory system

Example: lepton-proton scattering
(neutrino, muon, electron beams off stationary target)

Lepton beam: $p_l=(E_l,0,0,k_z)$

$p_l^2=m_l^2=E_l^2-k_z^2 \rightarrow E_l \approx |k_z|$

$p_l=(E_l,0,0,k_z)$

Proton fixed target: $p_p=(E_p,0,0,0)$

$p_p^2=E_p^2=M_p^2$

$p_p=(M_p,0,0,0)$

$\text{Energy}^2=s=(p_l+p_p)^2=m_l^2+M_p^2+2E_lM_p \approx 2E_lM_p$

Example: lepton-proton collider:

$p_l=(E_l,0,0,k_{z,l})$, $k_{z,l}^2=E_l^2-m_l^2$

$p_p=(E_p,0,0,k_{z,p})$ $k_{z,p}^2=E_p^2-M_p^2$

[Centre of mass system: $k_l=-k_p$]

$s=m_l^2+M_p^2+2E_lE_p-2\cos(k_l,k_p)k_lk_p$

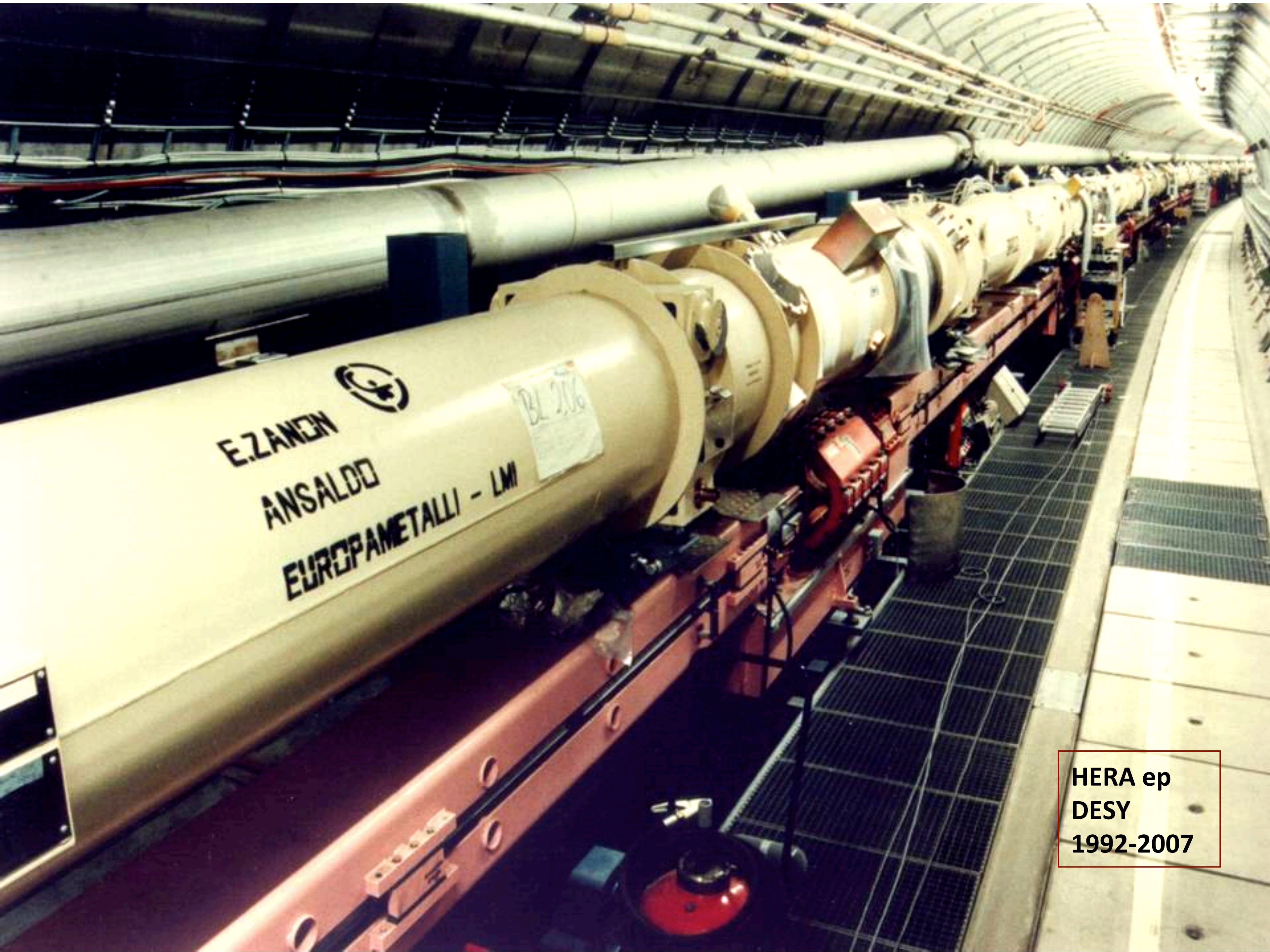
$s=m_l^2+M_p^2+2E_lE_p+2k_{z,l}k_{z,p} \approx 4E_lE_p$

HERA: E_e = 27.5 GeV E_p = 920 GeV

$s=1.01\ 10^5\ \text{GeV}^2$

equivalent fixed target energy:
$E_l^* = 2E_eE_p/M_p = 53.9\ 10^3$ GeV = 53.9 TeV

Particle Physics needs colliders to reach highest energies
Famous hadron-hadron colliders: (ISR, SPS, Tevatron, LHC, ?)



HERA ep
DESY
1992-2007

Cross Section

→ |.....|

beam I_0 target z

density $\rho = n/Vol$

N interactions

$N \sim I_0\ \rho\ z = L$

$N = L\ \sigma\ A$

L - Luminosity

A- Acceptance of process
reconstruction - requires
Monte Carlo simulations!

σ - Cross section = $N/(A\ L)$

$I = I_0 \exp(-z/\lambda)$

I = number of beam particles
surviving without any interaction

$\lambda = 1/\rho\sigma$ mean free path length

$N = I_0 - I$

$N = I_0\ (1 - e^{-\rho\sigma z})$

for small $\rho\sigma z$:

$N = I_0\ \rho\ z\ \sigma$

Cross section = $1/(\rho\lambda)$

Cross Section Examples

$$[\sigma] = m^2$$

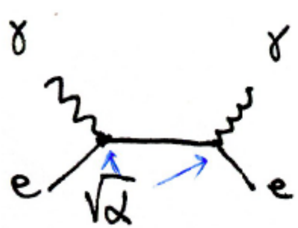
$$1barn = 10^{-28} m^2$$

$$[L] = m^{-2}$$

$$[L] = pb^{-1}$$

units

A cross section of 1pb when measured with a luminosity of 1pb^{-1} in a certain time interval leads to 1 scattering event.



Feynman Diagram for Thompson Scattering

$$\sigma = \frac{2}{3} \cdot \alpha^2 \cdot 4\pi R_e^2$$

Liquid hydrogen ionization bubble chamber (2m HBC, CERN)

Anti-proton beam of E=2.5 GeV

Photograph reveals 15 incoming tracks and 4 interactions, how large is the scattering cross section?

$$\rho = 4 \cdot 10^{28} \, p/m^3$$

$$\frac{N}{I_0} = 1 - e^{-\rho\sigma\cdot z}$$

$$-\ln(1 - \frac{N}{I_0}) = \rho\sigma \cdot z$$

$$z = 2m$$

$$N = 4$$

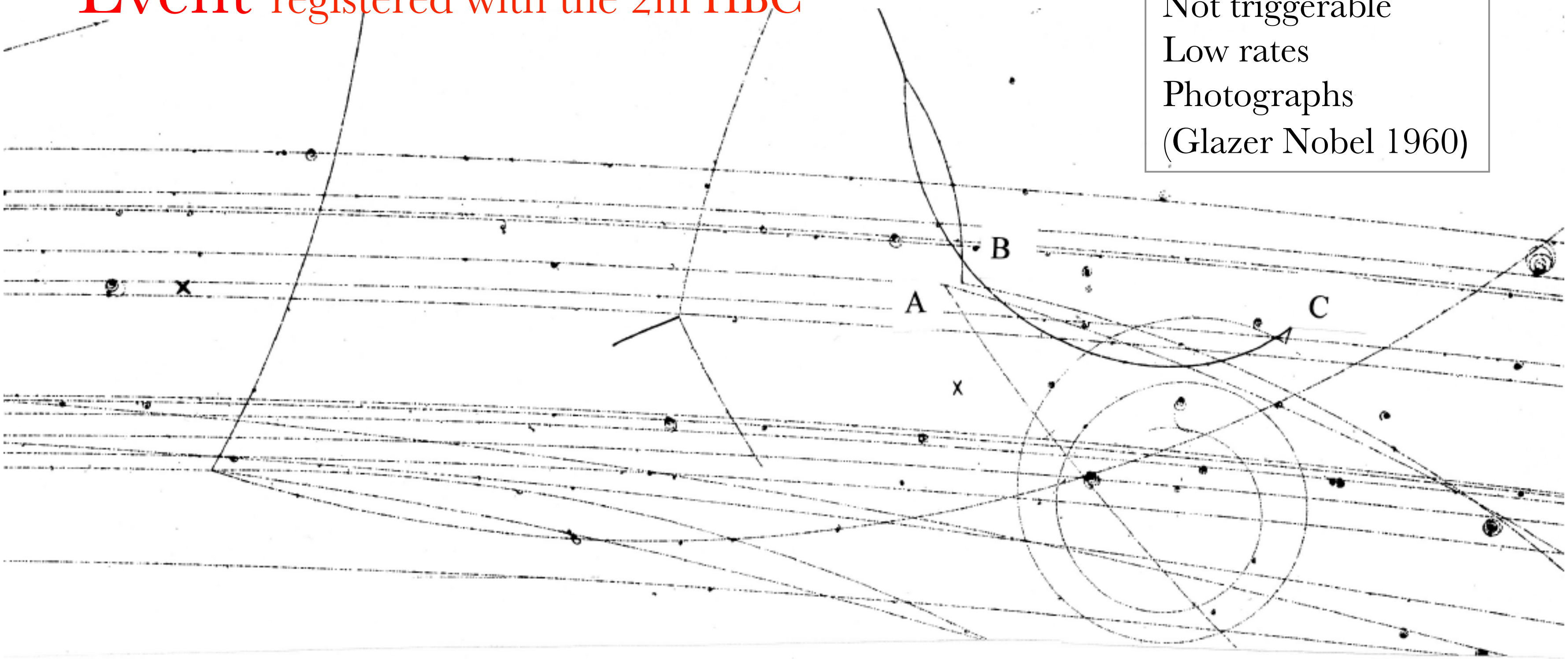
$$I_0 = 15$$

$$\sigma = 0.04 \cdot 10^{-28} m^2$$

calculation belongs to next picture →

Event registered with the 2m HBC

- Ionization
- High resolution
- Not triggerable
- Low rates
- Photographs
- (Glazer Nobel 1960)



Bubble Chamber Picture c 1966: Anti-protons, $\bar{p}$,of momentum 2.5 GeV/c travel through liquid hydrogen. The picture covers a length of ~1.5 m. At point **A** a $\bar{p}$ interacts with a proton to produce 2 charged mesons and a neutral K^0. This travels to point **B** where it decays $K^0 \rightarrow \pi^+\pi^-$. The π^+ travels upwards, bounces off a proton in the hydrogen and at point **C** decays $\pi^+ \rightarrow \mu^+\upsilon_\mu$. After travelling a few cms, the muon decays $\mu^+ \rightarrow e^+\upsilon_e\overline{\upsilon}_\mu$. The positron spirals around, loosing energy until it annihilates with an electron, $e^+e^- \rightarrow 2\gamma$, which leave the chamber.

M.Houlden

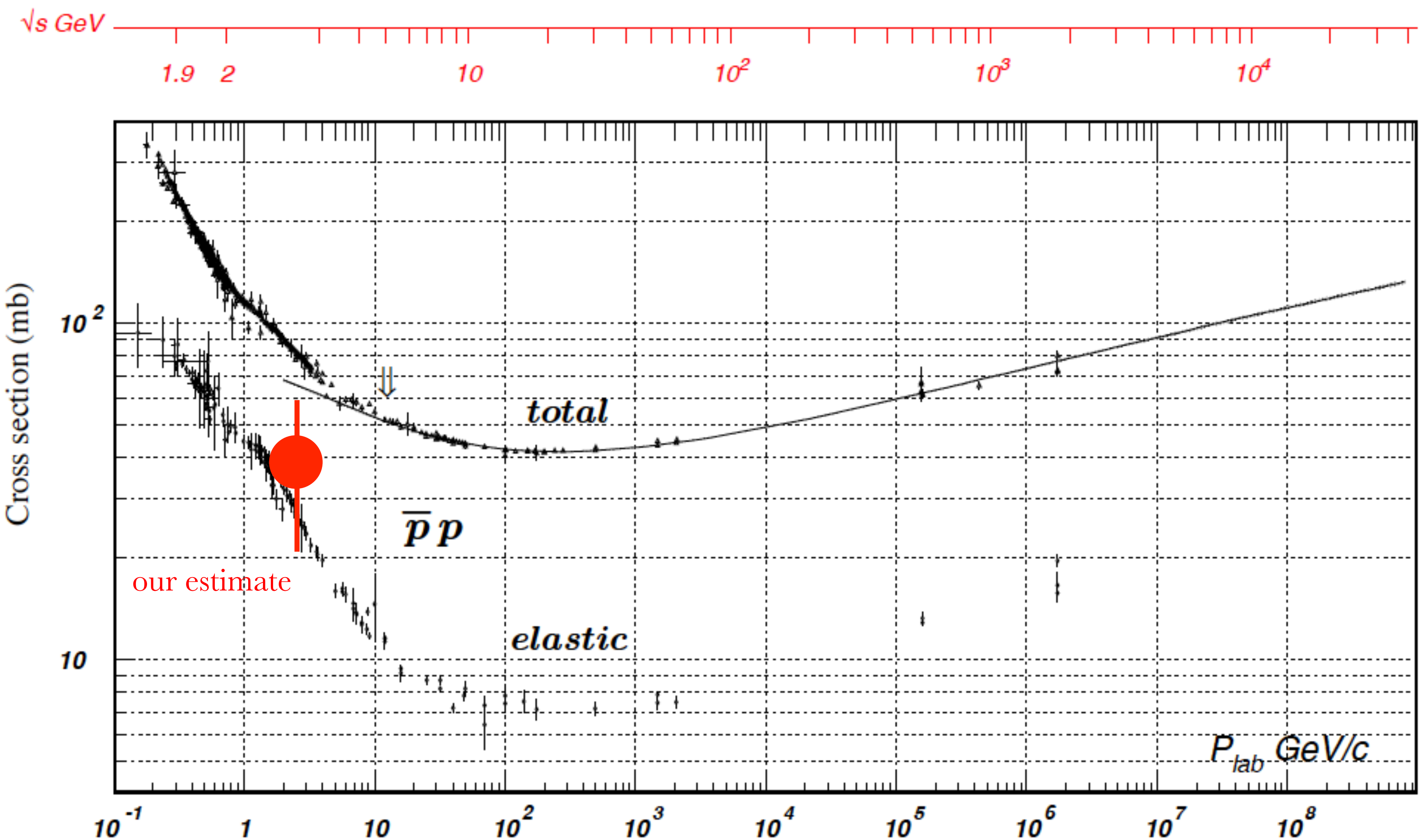
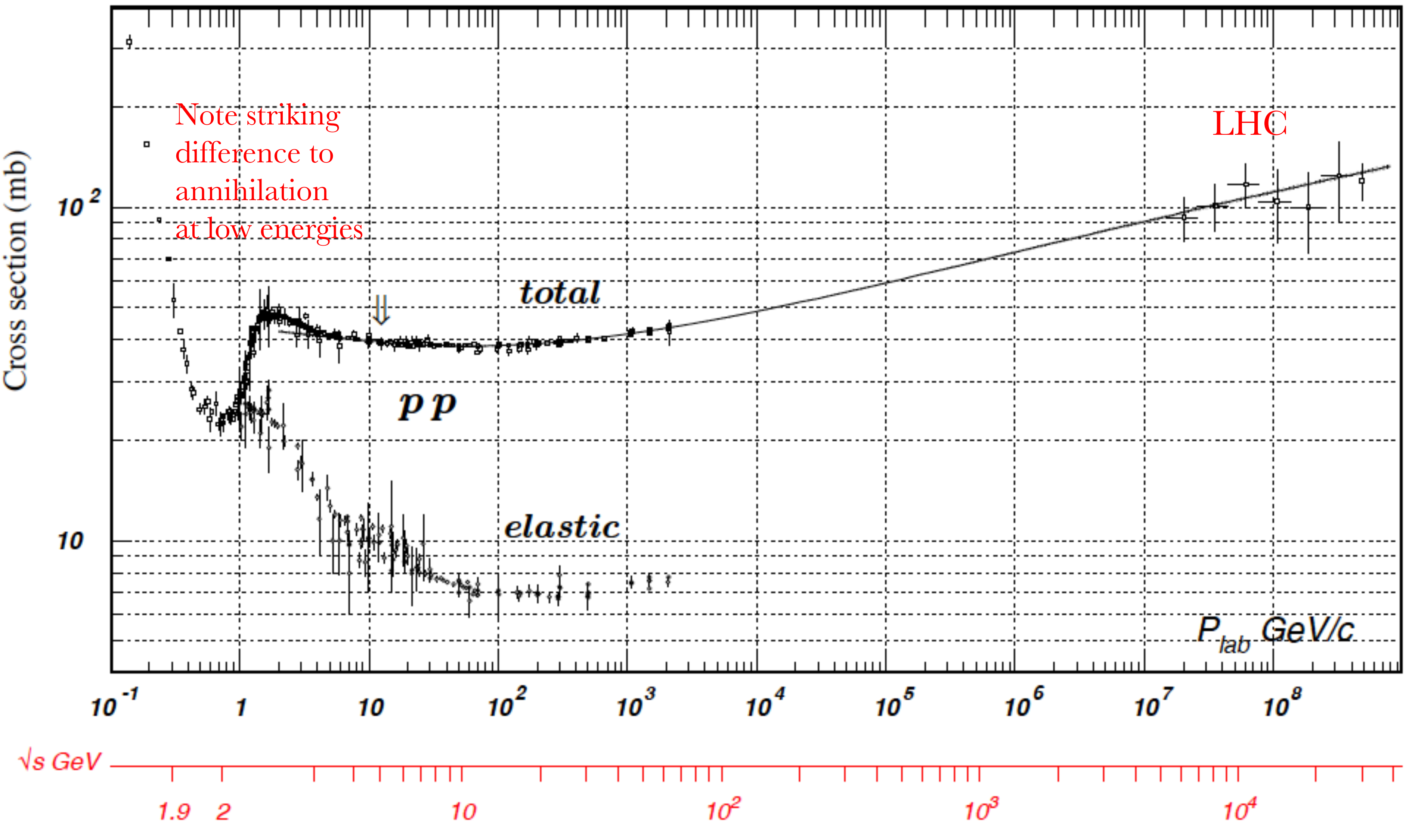


Figure 41.11: Total and elastic cross sections for pp and $\bar{p}p$ collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at `http://pdg.lbl.gov/current/xsect/`. (Courtesy of the COMPAS group, IHEP, Protvino, August 2005)

With enlarged statistics get accurate measurements



Note striking
difference to
annihilation
at low energies
LHC
total
p p
elastic
Cross section (mb)
10^2
10
P_{lab} GeV/c
10^{-1}
1
10
10^2
10^3
10^4
10^5
10^6
10^7
10^8
√s GeV
1.9
2
10
10^2
10^3
10^4

Decay Width and Branching Ratio

Decay width

$$\Gamma = \frac{\hbar}{\tau}$$ lifetime at rest

$$\Gamma = \frac{1}{\tau}$$

$$\Gamma = \sum_{f=1}^{n} \Gamma_f$$ n decay channels

Heavy quark states decay after O(300) µm → modern particle physics experiments are equipped with Silicon strip (or pixel) detectors of typically 20 (5) µm resolution to detect heavy particles and for precision tracking near the interaction point.

Branching ratio

$$b_f = \frac{\Gamma_f}{\Gamma} = \frac{\tau}{\tau_f}$$

$$\mu^- \to \nu_\mu e^- \overline{\nu_e}$$ **b=100%**

$$\tau^- \to \nu_\tau e^- \overline{\nu_e}$$ **b= 17%**

Lifetime

long lived: > 10^{-16}s
Measure decay lengths with high resolution detectors

short lived: determine width of resonant state from invariant mass distribution of decay particle momenta → lieftime

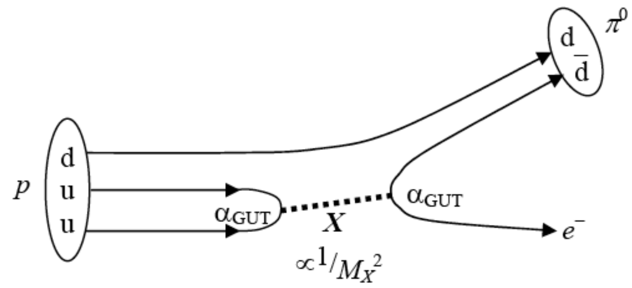
Lifetime τ

Interaction	lifetime [s]
Strong	$10^{-23\pm1}$
Electromagnetic	$10^{-19\pm2}$
Weak	$10^{-10\pm3}$

neutron β decay: $n \rightarrow p\, e^- \bar{\nu}_e$

neutron lifetime is (885.7 ± 0.8) s

Is the proton a stable particle (?)



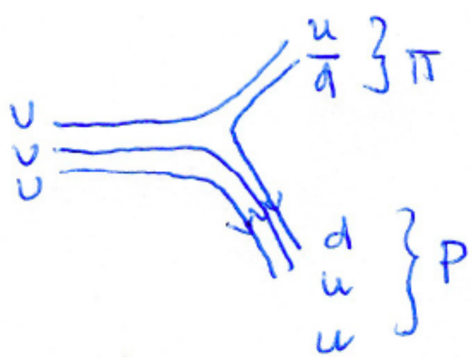
$$\tau_p^{-1} = \Gamma(p \rightarrow e^+\pi^0) \approx \frac{\alpha_{GUT}^2 m_p^5}{M_X^4}$$

In some theories
(grand unified theories GUTs)
the proton is not stable.

A lifetime of order
10^{35} years corresponds
to a mass scale of order
10^{16} GeV - the "Planck mass"

Resonances

$\Delta^{++} \Rightarrow \pi^{+} p$



quark decay diagram

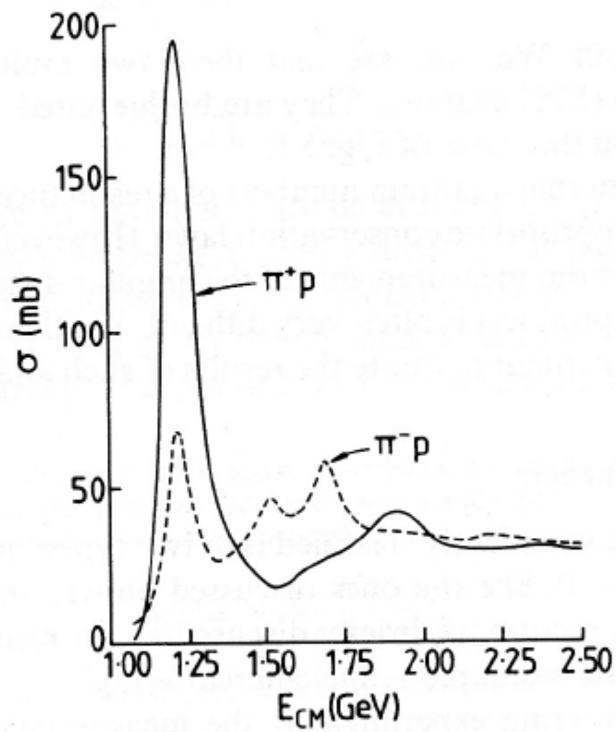
The delta resonance decays in about 10^{-23}s

$E_{cm}^2 = M^2(\pi p) = (p_\pi + p_p)^2$

Colour degree of freedom:

Pauli statistics: $u_r u_b u_g$

Production of the Δ resonance in pion-proton scattering ("formation").
M(Δ) = 1.232 GeV



$$P_f(E) = \frac{1}{2\pi} \cdot \frac{\Gamma_f}{(E-M)^2 + \Gamma^2/4}$$

Breit-Wigner formula to determine width (Γ=1/τ)

cf B.Martin, G.Shaw "Particle Physics"

Summary

1. Kinematics determines event configuration.
2. Conservation of energy, 3-momentum, 4-momentum
3. $P=(E,p)$, $P^2=M^2-p^2$
4. Highest energies in accelerators are reached with colliders, luminosity a challenge.
5. The observed number of events is proportional to the luminosity, and $\sigma=N/LA$
6. The strong, electromagnetic and weak interactions have increasing lifetime $10^{-24\,..-13}$s.
7. Typical weak decay lengths of 200μm have lead to a revolution of tracking (Silicon!)
8. The proton is stable, we think, $\tau > 2\ 10^{29}$ years. The neutron decays after 886s.
9. Hadronic resonances such as the Δ decay after 10^{-23}s (reconstruction through decays).

History of Particle Physics could be taught as a sequence of spectroscopies,
but no sub-quark or high mass (SUSY) spectroscopy has yet been discovered..

Particle Physics - a Sequence of Spectroscopies

- "Excitation of the 2536 Å Resonanc Line of Mercury"
  Franck /Hertz 1914
  Bohr $\rightarrow$ <u>ATOMIC SPECTROSCOPY</u>

- "Disintegration of Elements by High Velocity Protons"
  Cockcroft / Walton 1932
  pLi $\rightarrow \alpha\alpha$: <u>NUCLEAR SPECTROSCOPY</u>

- "Total Cross-Sections of Positive Pions in Hydrogen"
  Anderson/Fermi/Long/Nagle 1952
  $\Delta^{++} \rightarrow p\pi$: <u>HADRON SPECTROSCOPY</u>

- The charming "November Revolution"
  Ting et al., Richter et al. 11.11.1974
  $\mathcal{J}/\Psi \rightarrow c\bar{c}$: <u>QUARK SPECTROSCOPY</u>

Gustav Hertz: Nobel 1925


John Cockroft and Ernest Walton: Nobel 1951


Enrico Fermi: Nobel 1935


Sam Ting and Burt Richter: Nobel 1976