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A Determination of the Leptonic Neutral Current Couplings

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Abstract. An analysis is presented of the recent data which are sensitive to the e, μ and τ neutral current couplings. A fit combining all results ($e^+e^-, \mu C, \nu e, eD$, atoms) selects a unique solution in agreement with the standard-model expectation. Assuming lepton universality, the vector and axial-vector couplings are determined to be $v = -0.013 \pm 0.048$ and $a = -0.520 \pm 0.014$. Similarly we find ($\sin^2\theta = 0.213 \pm 0.012$, $\rho = 1.015 \pm 0.038$) or ($\sin^2\theta = 0.211 \pm 0.012, \rho \equiv 1$) which, combined with all other values, gives an average of $\sin^2\theta = 0.216 \pm 0.006$.

1. Introduction

It has been the aim of many neutral current experiments [1] during the last years to determine the coupling constants of various elementary fermions (f) to the Z_0-boson. In a general $SU(2) \times U(1)$ theory the vector and axial-vector couplings are defined as

$$v_f = I_3^L(f) + I_3^R(f) - 2Q_f \sin^2\theta, \quad a_f = I_3^L(f) - I_3^R(f) \tag{1}$$

with $I_3^{L(R)}$ the left-handed (r.h.) weak isospin charges, Q_f the electric charge and θ the Weinberg angle. Due to recent experimental progress in the leptonic sector it becomes now possible to uniquely determine all (v, a) lepton couplings, apart from v_τ. The basic aim of this paper is to use the available neutral current data for a consistent and simultaneous determination of the lepton couplings, which so far has not been undertaken [2, 3]. This yields additional constraints on the standard-model parameters ρ and $\sin^2\theta$ as well.

2. Data Summary and Treatment

The following types of experiments are sensitive to some lepton couplings:

i) parity violating transition amplitudes in heavy atoms are proportional to a_e. These data are now consistent with each other and becomes more useful numerically. For our purpose they determine the weak charge

$$Q_w = 2a_e(N - Z(1 - 4\sin^2\theta_h)) \tag{2}$$

where θ_h denotes the mixing angle entering the hadronic current. We have used data for Bi, Tl, Cs [4] and Pb [5] adding the still sizeable theoretical uncertainties ($\sim 25\%$) in quadrature to the experimental ones.

ii) the asymmetry measured at SLAC in polarized eD scattering [6] is sensitive to a parity violation combination of vector and axial-vector coupling according to

$$A^-/Q^2 = \kappa(a_e V - v_e A_0 g(y)) \tag{3}$$

with $V = 1.2(2v_u - v_d), A_0 = 1.2(-2a_u + a_d)/(1 + \xi)$ and $g(y) = (1 - (1-y)^2)/(1 + (1-y)^2)$. Here ξ denotes the ratio of sea to valence quark distributions which was calculated using the parametrizations of [7]. The resulting correction is known to be a small effect only because of $v_e \cdot g(y) \leqq 0.05$ in the kinematic region of the experiment. The size of the γZ interference effects is given by the parameter $\kappa = G/\sqrt{2} 2\pi\alpha$ with G the Fermi constant and α the fine structure constant. Using Q_w and A^- to get v_e and a_e requires to preset $\sin^2\theta_h$ in order to calculate the hadronic vector current contribution. We assumed $\sin^2\theta_h = 0.224 \pm 0.012$, the recent average value from deep inelastic neutrino scattering [8] multiplied by 1.006 [9]. The $\sin^2\theta_h$ uncertainty has been included into the resulting errors.

iii) elastic neutrino–electron scattering [10]. Recent v_μ data represent a serious constraint for our analysis. The cross-sections at given energy E are described by

$$\sigma/E = \frac{G^2 m_e}{2\pi}[(v_e \pm a_e)^2 + \tfrac{1}{3}(v_e \mp a_e)^2] \tag{4}$$

where m_e is the electron mass. For the $\bar{v}_e e$ reactor data we used the formula of [11] with the coefficients readjusted according to [2].

iv) The $\mu^\pm C$ asymmetry measurement of the BCDMS collaboration [12] determines a combination of muon couplings according to

$$B = \frac{\sigma^+(-\lambda) - \sigma^-(+\lambda)}{\sigma^+(-\lambda) + \sigma^-(+\lambda)} = -\kappa(a_\mu - \lambda v_\mu) \cdot A_0 \cdot g(y) \cdot Q^2 \quad (5)$$

where λ is the longitudinal muon beam polarization.

v) The $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$ asymmetry data from PEP and PETRA [13] provide us essentially with $a_e a_\mu$ and $a_e a_\tau$, although we have included the κ^2 contributions containing the vector couplings as well, i.e. the forward–backward asymmetry is

$$A_{FB} = -\tfrac{3}{2}\kappa a_e a_\mu \cdot s \cdot \frac{M_z^2}{M_z^2 - s} \cdot \frac{1 - \kappa \cdot 2 v_e v_\mu}{1 - \kappa 2 v_e v_\mu + \kappa^2 (v_e^2 + a_e^2)(v_\mu^2 + a_\mu^2)} \quad (6)$$

For the Z_0-mass we assumed $M_Z = (93.0 \pm 2.0)$ GeV based on recent UA1, 2 results [14] and included δM_Z into the resulting errors. We have disregarded Bhabha scattering data results as they are still less significant [15].

A consistent treatment of the data requires to correct for electroweak second order effects. In the on-mass shell renormalization scheme [9] electroweak radiative corrections are almost completely absorbed into a redefinition of α. We have correspondingly modified the κ value (3) by a factor 0.9304 [16] in order to account for these effects in the γZ asymmetry data. Further radiative contributions due to the energy and process variations amount to a few per cent of the correction which is negligible compared to the present experimental errors. Note for example that even the precise A^- data determine $\sin^2\theta$ only at the 5% level [17]. Restricting the corrections to a redefinition of α implies the assumption that the present νe data can be considered to be free of electro-weak corrections. This approximation is justified by detailed calculations [18]. Similarly, recent evaluations of electroweak corrections for the e^+e^- asymmetry data find a factor of about 0.93 for the effect of the one-loop corrections on the lowest order asymmetry at PETRA energies [19]. For the atomic data use has been made of the corrected Q_w expression, (2), as calculated in [21]. Thus all subsequent results can be considered to be related to the on-mass shell renormalization scheme. Whenever needed, statistical and systematic errors have been added in quadrature.

3. Fit Results

For the derivation of results a MINUIT [23] fitting procedure has been used minimizing the χ^2 based on a sum over all data. A five-parameter fit uniquely determines the v and a couplings to be

$$\begin{aligned} v_e &= -0.033 \pm 0.059 \quad v_\mu = -0.103 \pm 0.172 \\ a_e &= -0.501 \pm 0.031 \quad a_\mu = -0.587 \pm 0.052 \\ a_\tau &= -0.474 \pm 0.076 \end{aligned} \quad (7)$$

Table 1. Correlation coefficients for general (v, a) fit

	v_e	a_e	v_μ	a_μ	a_τ
v_e	1				
a_e	−0.53	1			
v_μ	0.12	−0.22	1		
a_μ	0.31	−0.56	0.37	1	
a_τ	0.16	−0.31	0.07	0.19	1

Table 2. Correlation coefficients for $(\rho, \sin^2\theta, I_3^R)$ fit

	ρ	$\sin^2\theta$	$I_3^R(e)$	$I_3^R(\mu)$	$I_3^R(\tau)$
ρ	1				
$\sin^2\theta$	0.65	1			
$I_3^R(e)$	−0.95	−0.50	1		
$I_3^R(\mu)$	−0.18	−0.26	0.01	1	
$I_3^R(\tau)$	−0.02	−0.05	−0.08	0.06	1

with a χ^2 per degree of freedom (χ_D^2) of 0.75. The errors quoted for multidimensional fits define the one-standard deviation for a given parameter independently of the others [23]. The correlation matrix for (7) is given in Table 1. The v and a values are in very good agreement with the standard-model predictions (1) $v_e = v_\mu = -.06$ at $\sin^2\theta = 0.22$ and $a_e = a_\mu = a_\tau = -1/2$. Yet, one still misses v_τ and a more accurate v_μ. Natural current lepton universality is confirmed also by an equivalent determination of ρ, $\sin^2\theta$ and the r.h. weak charges yielding

$$\begin{aligned} \rho &= 0.80 \pm 0.12 \qquad \sin^2\theta = 0.19 \pm 0.02 \\ I_3^R(e) &= 0.13 \pm 0.09 \qquad I_3^R(\mu) = 0.09 \pm 0.05 \\ I_3^R(\tau) &= -0.03 \pm 0.08 \end{aligned} \quad (8)$$

Note that here $\sin^2\theta_h$ (2, 3) has been considered as a free parameter. The correlation matrix (Table 2) reveals a strong negative correlation between $\rho, \sin^2\theta$ and $I_3^R(e)$ which means that the somewhat high $I_3^R(e)$ value compensates for the rather low values of ρ and $\sin^2\theta$ (see below). From this joint fit the existence of r.h. doublets, i.e. $I_3^R = +1/2$, is excluded at the level of 4, 8, 6 standard deviations for e, μ and τ respectively. The I_3^R errors can be reduced if ρ and $\sin^2\theta$ are kept constant. For $\rho = 1$ and $\sin^2\theta = 0.22$, for example, we find $I_3^R(e) = 0.06 \pm 0.02$, $I_3^R(\mu) = 0.07 \pm 0.04$ and $I_3^R(\tau) = -0.03 \pm 0.08$. Assuming lepton universality the r.h. weak charge is determined to be zero with high precision, i.e. $I_3^R = .02 \pm .02$ at $\sin^2\theta = 0.22$ and $\rho = 1$.

Subsequently, e, μ and τ are assumed to have identical coupling constants v and a. A two-parameter fit to all data finds

$$v = -0.013 \pm 0.048 \quad a = -0.520 \pm 0.014 \quad (9)$$

with a χ_D^2 of 0.75 and a correlation coefficient of 0.37.

Table 3. Summary of (v, a) fits assuming $v_e = v_\mu, a_e = a_\mu = a_\tau$, $\sin^2\theta = 0.223$ $M_Z = 93.0\,\mathrm{GeV/c}$

	v	a	χ_D^2
All data	-0.013 ± 0.048	-0.520 ± 0.014	0.75
no νe	-0.082 ± 0.094	-0.516 ± 0.016	0.88
no e^+e^-	-0.028 ± 0.050	-0.503 ± 0.024	1.04
only leptonic data	0.011 ± 0.049	-0.529 ± 0.015	0.43
νe only (v_e, a_e)	0.011 ± 0.052	-0.529 ± 0.035	0.28

We have excluded one by one the more accurate data sets $(\nu e, eD, e^+e^-)$ and find always similar central values though with differing accuracy, see Table 3. These fits are illustrated in Fig. 1a presenting 90% confidence level contours in the (v, a) plane. The two solutions of the neutrino data (dashed-dotted) are resolved by any of the other experiments. Fitting the νe and the e^+e^- data together, i.e. using the leptonic data only, we essentially reduce the error of a about twice (dashed curve in Fig. 1a, Table 3). The consideration of the "hadronic data" (atoms, $eD, \mu C$) is seen to have only a slight influence on the (v, a) contour leading to the shadowed central region.

Let us finally turn to a determination of the ρ parameter and $\sin^2\theta$ assuming $I_3^R = 0$. A two-parameter fit to all data yields

$$\rho = 1.015 \pm 0.038 \quad \sin^2\theta = 0.213 \pm 0.012 \qquad (10)$$

with $\chi_D^2 = 0.77$ and a correlation coefficient of 0.37. These numbers are in remarkable agreement with recent νN and $\bar{p}p$ results [13, 16]. Contrary to the (v, a) contour, for $(\rho, \sin^2\theta)$ the hadronic data considered here are important. This is due to the fact that the eD asymmetry essentially determines $\sin^2\theta$ which explains the slight shift and reduction of the $(\rho, \sin^2\theta)$ contours due to the (νe) and leptonic data only (see Fig. 1b).

The superposition of all data apart from e^+e^- yields $\sin^2\theta = 0.211 \pm 0.011$ in the on-mass shell scheme setting ρ to be one. Rewriting the κ factor as

$$\kappa = \frac{1}{4\sin^2\theta\cos^2\theta M_Z^2} \qquad (11)$$

allows to derive a $\sin^2\theta$ measurement from the e^+e^- asymmetry data as well. Using (11) leads to negligible electroweak higher-order corrections to A_{FB} at PETRA energies, i.e. this factor has not been multiplied by 0.93. Note that independently of the way κ is expressed, the theoretical predictions for A_{FB} agree at the one-loop level although the Born term asymmetries differ from each other [20]. Using the recent data set including $A_{FB}(\tau)$ we find $\sin^2\theta = 0.186 \pm 0.021$ in good agreement with the original result [22].

Combining these two values with $\sin^2\theta$ from νN scattering as quoted above [8] and with the $\sin^2\theta$ values from the W mass measurements [13] we find for the weighted average $\sin^2\theta = 0.216 \pm 0.006$. Treating

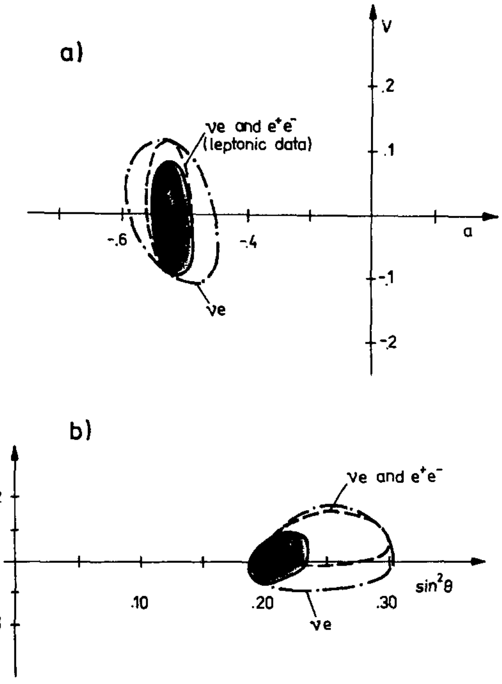

Fig. 1a and b. 90% confidence level contours for two-parameter fits to neutral current data for: **a)** vector—and axial-vector leptonic couplings, **b)** ρ and $\sin^2\theta$. Dashed curves: $\bar{\nu}_\mu e$ and $\bar{\nu}_e e$ (The νe data yield a second solution $(v \sim -0.5, a \sim 0.0)$ not shown here); Solid curves: Leptonic data $(\nu e, e^+e^-)$; Shadowed region: all data (atoms, $eD, \mu C$ and leptonic)

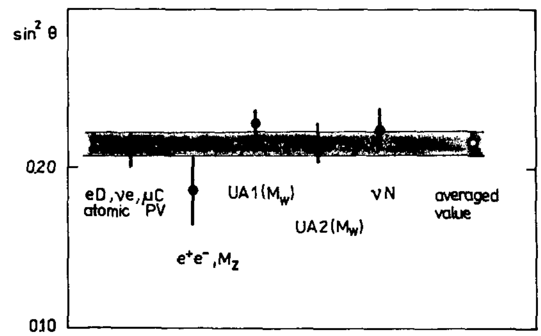

Fig. 2. $\sin^2\theta$ determinations in the on-mass shell renormalization scheme using data from $(eD, \nu e, \text{atoms}, \mu C)$, $(e^+e^-$ and $M_Z)$, UA1 and UA2 [14] and νN [8]. The error bars are the combined statistical and systematic errors

systematic and statistical errors separately yields $\sin^2\theta = 0.219 \pm 0.004\,(\text{stat}) \pm 0.010\,(\text{syst})$. Figure 2 displays all $\sin^2\theta$ measurements which are in remarkable agreement with each other. The central value is very close to the $SU(5)$ prediction $\sin^2\theta = 0.215 \pm 0.003$ [23] at $\Lambda_{MS} = 160$ MeV. Future single experiments will achieve similar accuracies which should allow to precisely test the standard model at the $O(\alpha)$ level. Simultaneously, these experiments will yield the lepton couplings with much improved precision.

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