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Electroweak Interactions Probing the Nucleon Structure

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Abstract. The possibility is studied of isolating single electroweak structure functions in deep inelastic lepton–nucleon scattering at Q^2 around M_Z^2. Utilizing the cross-section dependence on the lepton charge and longitudinal polarization λ it is shown that the most-conclusive results follow from measurements of $\sigma^+(-\lambda) \pm \sigma^-(+\lambda)$. Considerable simplifications of the problem are possible if λ can be tuned to $\sin^2\theta$ dependent values of about 0.2. New quark–parton model relations are derived.

1. Introduction

Deep inelastic lepton–nucleon scattering has been fundamental in the understanding of the nucleon structure [1]. Presently a new generation of muon fixed target [2] and electron–proton collider experiments [3] is being discussed. At high momentum transfers Q^2 comparable to M_Z^2, electroweak interference and pure weak effects make a substantial contribution to the scattering cross section. For unpolarized nucleons this implies the occurrence of 8 structure functions [4], i.e. F_1, F_2 from the one-photon exchange cross section σ_0, G_1, G_2 and xG_3 from the γZ interference part σ_i and H_1, H_2 and xH_3 from the pure weak part σ_z. Due to the large number of structure functions, the contributions from σ_i and σ_z have been regarded as an obstacle to the investigation of the nucleon structure at highest Q^2. Therefore, quantitative tests of QCD, in particular at collider energies, have been foreseen only up to $Q^2 = 1000$ $(\mathrm{GeV/c})^2$ [3] where the one-photon exchange approximation still holds to about 20%. This implies, however, vasting the high energy reached. The Q^2 region above $1000\,(\mathrm{GeV/c})^2$ has been considered to be studied by $\ell N \to \nu_\ell X$ scattering [3] which depends on 3 structure functions only. However, this measurement relies on a high resolution detection of the hadronic shower direction and energy.

Of course, it is desirable to explore the highest Q^2 region also with data from the neutral-current reaction $\ell N \to \ell X$. This can be discussed in a simplified way introducing additional assumptions on the nucleon substructure [5, 6] which effectively replaces the framework of structure functions by that of parton distributions. Alternately, here possibilities are investigated to separate single structure functions as the basic quantities of interest. To some extent this turns out to be feasible indeed, utilizing the lepton charge and helicity dependence of the cross section. In this more general approach one only has to assume that the leptonic vertex is correctly described by the GWS theory.

The subject is organized as follows: Section 2 contains relevant formulae for cross sections and structure functions. In Sect. 3 structure-function measurements in the interference region are discussed i.e. up to $Q^2 \sim 6000\,(\mathrm{GeV/c})^2$ where the pure weak contribution σ_Z is negligible. That part is included in Sect. 4 where particular attention is paid to the lepton polarization dependence of the cross section. Section 5 presents some quark–parton model (QPM) relations for protons and isoscalar targets in order to illustrate the physics information contained in the electroweak structure functions. Appendix A contains a systematic consideration of cross-section sums and differences resulting from the inherent charge and helicity dependences.

2. Definitions

The double differential cross section for deep inelastic polarized lepton scattering off unpolarized nucleon targets can be represented as

$$\frac{d\sigma}{dQ^2\,dx}(\ell^\pm(\lambda) N \to \ell^\pm X) = \sigma^\pm(\lambda) = \sigma_0 + \sigma_i^\pm(\lambda) + \sigma_z^\pm(\lambda) + \sigma_{\mathrm{Rc}} \tag{2.1}$$

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where λ is the helicity of the beam and σ_0, σ_i, σ_z are the one-photon exchange, the γZ interference and the Z-boson exchange cross sections, respectively. σ_{Rc} denotes higher-order radiative corrections which have been calculated up to TeV energies [7].

The GWS theory has 3 independent parameters which have to be selected out of $\alpha, G, \sin^2\theta, M_W$ and M_Z. We have chosen the parameters used in the on-mass shell renormalization scheme [8], i.e. the fine-structure constant α and the physical boson masses $M_W = 83.0$ GeV and $M_Z = 93.8$ GeV. This fixes $\sin^2\theta$ to all orders of perturbation theory to be

$$\sin^2\theta = 1 - M_W^2/M_Z^2 = 0.217 \tag{2.2}$$

Furthermore, in the tree approximation we get $G/\sqrt{2} = \pi\alpha/(2\sin^2\theta \cdot M_W^2)$ which is numerically slightly different from the Fermi constant G_F measured at $Q^2/M_W^2 \ll 1$.

Rewriting σ as $\sigma \cdot Q^4 x/(2\pi\alpha^2)$, the explicite expressions (see e.g. [3,L1]) for the cross sections of (2.1) are*

$$\sigma_0 = [Y_+ \cdot F_2(x, Q^2)] \tag{2.3}$$

$$\sigma_i^{\pm}(\lambda) = \kappa(Q^2) \cdot [Y_+(-v \mp \lambda a) G_2(x, Q^2) + Y_-(\pm a + \lambda v) x G_3(x, Q^2)] \tag{2.4}$$

$$\sigma_z^{\pm}(\lambda) = \kappa^2(Q^2) \cdot [Y_+(v^2 + a^2 \pm 2va\lambda) H_2(x, Q^2) + Y_-(\mp 2va - (v^2 + a^2)\lambda) x H_3(x, Q^2)] \tag{2.5}$$

Here $v = I_3^L - 2Q\sin^2\theta = 0.06$ and $a = I_3^L = -1/2$ are the weak neutral current couplings of the leptons and $Y_{\pm} = 1 \pm (1-y)^2$. x and y are the familiar scaling variables.

The relative magnitudes of the cross sections (2.3–2.5) are governed by $(1, \kappa, \kappa^2)$, respectively, where

$$\kappa(Q^2) = \kappa(\infty) Q^2/(Q^2 + M_Z^2) \tag{2.6}$$

$$\kappa(\infty) = [4\sin^2\theta\cos^2\theta]^{-1} = 1.47 \tag{2.7}$$

They are displayed in Fig. 1.

Inverting (2.6) one finds that $\kappa = 1$ at $Q^2 = 18700 (\mathrm{GeV/c})^2$ which is the momentum transfer where all three cross sections are of comparable size**).

For later use we list the quark-parton model expressions of the electroweak structure functions which are

$$(F_2, G_2, H_2) = x\Sigma(q + \bar{q}) \cdot (Q_q^2, 2v_q Q_q, v_q^2 + a_q^2) \tag{2.8}$$

and

$$(xG_3, xH_3) = 2x\Sigma(q - \bar{q}) \cdot (a_q Q_q, a_q v_q) \tag{2.9}$$

In the GWS theory these functions are positive and of the same order of magnitude for $\sin^2\theta \approx .2$ with

* More precisely, F_2 should be replaced by $F_2 + (2xF_1 - F_2) y^2/Y_+$. A separation of F_1 and F_2 requires to vary the incoming energy and is beyond the aim of this study. This is true also for $G_{1,2}$ and $H_{1,2}$

**For small Q^2 this function behaves as $\kappa(Q^2 \ll M_Z^2) = 1.67 \cdot 10^{-4}$ $Q^2/(\mathrm{GeV/c})^2$. At low energies κ is usually defined by $Q^2 G_F/(\sqrt{2}2\pi\alpha) = 1.79 \cdot 10^{-4} Q^2/\mathrm{GeV/c})^2$. The numerical difference of both values can be traced back to higher-order corrections in the GWS theory

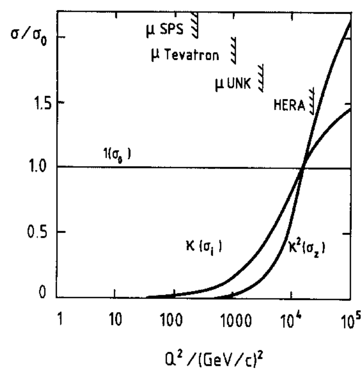

Fig. 1. Strength of σ_0, σ_i and σ_Z relative to $\sigma_0 vs Q^2$. κ is defined in (2.6), (2.7)

the neutral-current quark couplings given by $v_q = I_3^L(q) - 2Q_q\sin^2\theta$ and $a_q = I_3^L(q) = \pm\frac{1}{2}$.

3. Structure–Function Measurements up to $Q^2 \sim 6000\ (\mathrm{GeV/c})^2$

Let us firstly discuss the problem of isolating single structure functions at lower $Q^2 (< 6000 (\mathrm{GeV/c})^2)$ where the pure weak part involving H_2, xH_3 contributes to less than $\approx 10\%$. Thus, in this Q^2 region the cross section is sensitive mainly to F_2, G_2 and xG_3, see (2.3), (2.4). The cross section analysis can be simplified further by adjusting the lepton beam polarization to

$$\lambda(\ell^{\pm}) = \mp v/a = \mp (1 - 4\sin^2\theta) \approx \mp 0.13 \tag{3.1}$$

which completely removes the G_2 contribution from $\sigma_i^{\pm}$ (2.4).

Without the G_2 term the cross section becomes

$$\sigma^{\pm}(\lambda = \mp v/a) = Y_+ F_2 \pm Y_-(a - v^2/a)\kappa(Q^2) x G_3 \tag{3.2}$$

where again we have included $Q^4 x/(2\pi\alpha^2)$ in the definition of σ. One should note that this expression strongly resembles the cross section for charged-current neutrino nucleon scattering. In particular, the interference structure function xG_3(2.9) plays the role of xF_3^{ν}. Unfortunately, there seems to be no simple method to isolate G_2 even at Q^2 below $\lesssim 6000 (\mathrm{GeV/c})^2$.

4. Structure–Function Measurements at $Q^2 \sim M_Z^2$

Beyond the Q^2 region discussed, suitable combinations of structure functions can still be isolated utilizing the charge and polarization dependence of the cross section. In the Appendix we show that rather transparent results follow from two cross sections σ^+ and σ^- measured at opposite helicities (beam conjugation)*. The sum of these cross sections

$$B_+(\lambda) = \tfrac{1}{2}[\sigma^+(-\lambda) + \sigma^-(+\lambda)] \tag{4.1}$$

* We would like to remark that the beam conjugation has some advantages also for the detection of interference effects at low Q^2 in deep inelastic and elastic muon scattering [9]

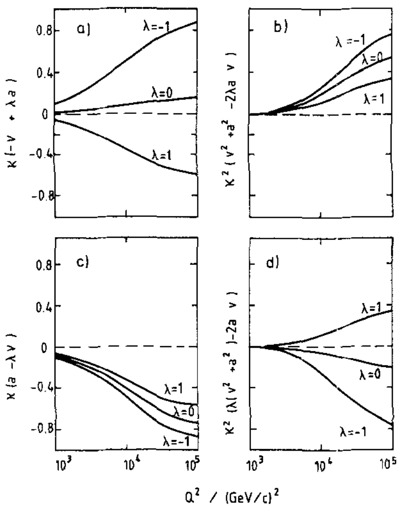

Fig. 2a–d. Factors appearing in-front of G_2, H_2 **a, b** and xG_3, xH_3 **c, d** in a measurement of $\sigma^+(-\lambda)+\sigma^-(+\lambda)$ and $\sigma^+-\sigma^-$, respectively. Curves are calculated in the GWS model, i.e. using $v=-1/2+2\sin^2\theta, a=-1/2$ and $\sin^2\theta=0.217$

is given by

$$B_+(\lambda)=Y_+[F_2+(-v+\lambda a)\kappa G_2 + (v^2+a^2-2va\lambda)\kappa^2 H_2] \tag{4.2}$$

This sum is proportional to $Y_+=1+(1-y)^2$ and contains only those functions which in the QPM involve $q+\bar{q}$ (2.8). Figures 2a, b illustrate the Q^2 dependence of the two λ dependent factors in (4.2) assuming $\sin^2\theta=0.217$. Once again, choosing

$$\lambda=v/a\approx 0.13 \tag{4.3}$$

removes the G_2 term from B_+ independently of Q^2. Thus $B_+(v/a)$ projects out the combination

$$B_+(v/a)=Y_+[F_2+(a^2-v^2)\kappa^2 H_2] \tag{4.4}$$

which below $Q^2\sim 6000(\mathrm{GeV/c})^2$ reduces to F_2.

The cross-section difference

$$B_-(\lambda)=\tfrac{1}{2}[\sigma^+(-\lambda)-\sigma^-(+\lambda)] = Y_-[(a-\lambda v)\kappa xG_3+(-2va+\lambda(v^2+a^2))\kappa^2 xH_3] \tag{4.5}$$

is proportional to $Y_-=y(2-y)$. Note that xG_3 and xH_3 are similar functions as they involve only $q-\bar{q}$ in the QPM (2.9). Figures 2c, d display the λ dependent factors of xG_3 and xH_3 for $\sin^2\theta=0.217$. It becomes evident from Fig. 2d that there is a possibility to remove the xH_3 contribution independently of Q^2 by choosing

$$\lambda=\frac{2va}{v^2+a^2} = \frac{2(1-4\sin^2\theta)}{1+(1-4\sin^2\theta)^2}\approx 0.26 \tag{4.6}$$

Then $B_-(\lambda)$ projects out the interference—structure function xG_3 only. Even if it is not possible to tune λ to the value (4.6), at Q^2 below $6000(\mathrm{GeV/c})^2$ the second term in (4.5) is negligible.

Probably, xG_3 is the structure function which may be measured best at high Q_2.

The first measurement of $xG_3(x)$ has been performed by the BCDMS collaboration at the SPS muon beam [10]. This experiment demonstrates the feasibility of measurements of structure functions beyond F_2 even at low $Q^2\sim 60(\mathrm{GeV/c})^2$. In the kinematic region of this experiment, the radiative corrections amount to about 40% of the electroweak contribution to B [11]. With rising Q^2 they become less important [7].

5. Some QPM Relations

It is not evident that the quark–parton model (or even QCD) will provide an adequate description of the nucleon structure in the Q^2 region discussed here. Nevertheless, the QPM is useful to illustrate the physics information contained in the electroweak structure functions.

The advantage of separating the $(q+\bar{q})$ from the $(q-\bar{q})$ functions via $B_\pm$ becomes quite transparent writing (4.2) and (4.5) for isoscalar targets:

$$B_+(\lambda)=Y_+f_+(Q^2,\lambda,M_z,M_W)\cdot x\Sigma(q+\bar{q})$$
$$f_+=(Q_u^2+Q_d^2)/2+(\lambda a-v)\kappa(v_uQ_u+v_dQ_d) + (v^2+a^2-2av\lambda)\cdot\kappa^2(v_u^2+a_u^2+v_d^2+a_d^2)/2 \tag{5.1}$$
$$B_-(\lambda)=Y_-f_-(Q^2,\lambda,M_z,M_W)\cdot x\Sigma(q-\bar{q})$$
$$f_-=(a-\lambda v)\kappa(a_uQ_u+a_dQ_d) + (\lambda(v^2+a^2)-2av)\kappa^2(a_uv_u+a_dv_d) \tag{5.2}$$

Obviously, (5.1) and (5.2) resemble the F_2 and xF_3 measurements in $\overset{(-)}{\nu}$ scattering with different kinematic factors.

As has been shown above, the most accurate measurements can be expected for F_2 and xG_3. For isoscalar targets (2.8) and (2.9) yield for four flavours

$$F_2^N=\tfrac{1}{2}(Q_u^2+Q_d^2)x\sum(q+\bar{q})-x(s-c)(Q_u^2-Q_d^2) \tag{5.3}$$

and

$$xG_3^N=(a_uQ_u+a_dQ_d)x\sum(q-\bar{q}) \tag{5.4}$$

In the valence-quark approximation the ratio xG_3/F_2 should be constant, i.e.

$$\frac{xG_3^N}{F_2^N}=\frac{2(a_uQ_u+a_dQ_d)}{Q_u^2+Q_d^2} \tag{5.5}$$

Neglecting the small term $\sim(s-c)$ for F_2, the antiquark distribution can be determined:

$$x\sum\bar{q}=\frac{9}{5}F_2^N-xG_3^N \tag{5.6}$$

Obviously, two independent cross-section measurements on isoscalar targets suffice to isolate $\sum\bar{q}$ or $\sum q$, respectively. The following relation can be derived from (5.1) and (5.2):

$$x\sum \bar{q} = c_+\sigma^+(-\lambda) + c_-\sigma^-(+\lambda) \tag{5.7}$$

with

$$c_\pm = \tfrac{1}{4}\left(\frac{1}{Y_+ f_+} \mp \frac{1}{Y_- f_-}\right)$$

and similarly for $\sum q$. Note that this relation is valid at arbitrary Q^2 and polarization values λ. Therefore, if λ can not be tuned, it would be very important to have a deuteron option in future ep colliders [3, 6] in order to unambiguously study the nucleon structure up to the kinematic or luminosity determined limits.

The proton structure functions may be used to determine e.g. the valence-quark distributions, i.e. for $x > 0.3$

$$xu_v = \frac{g}{2}F_2^p - \tfrac{3}{2}xG_3^p$$

$$xd_v = 6xG_3^p - gF_2^p \tag{5.8}$$

More generally, u_v and d_v can be written as linear combinations of $\sigma^+(-\lambda)$ and $\sigma^-(+\lambda)$ which are similar to (5.7). Therefore, quantities like the d/u ratio can be determined in charged lepton neutral current scattering using proton targets only. This is important because one gets rid off nuclear corrections like those due to Fermi motion and the "EMC effect" [12].

6. Summary

We have studied possibilities for isolating single structure functions in deep inelastic lepton–nucleon scattering at Q^2 around M_Z^2. In this region of high momentum transfers, 8 electroweak structure functions contribute to the neutral-current cross sections. The dependence of σ on the lepton charge and longitudinal polarization λ provides sensitivity to 5 functions: $F_2, G_2, xG_3, H_2, xH_3$ where F_2 should be replaced by $F_2(1+Ry^2/Y_+(1+R))$ if R differs from zero and similarly for G_2 and H_2. Considering combinations of cross sections we have shown that the most conclusive results can be derived from measuremens of $\sigma^+(-\lambda)$ and $\sigma^-(+\lambda)$ which allows to separate structure functions of different type, i.e. (F_2, G_2, H_2) from (xG_3, xH_3):

i) In the region Q^2 below $\sim 6000 (\text{GeV/c})^2$ the contribution from H_2 and xH_3 is negligible and σ is quite similar to the cross section for charged current neutrino–nucleon scattering. In particular, xG_3, following from the cross-section difference $\sigma^+ - \sigma^-$, resembles xF_3^ν.

ii) The sum $\sigma^+ + \sigma^-$ at arbitrary high Q^2 measures a combination of only F_2 and H_2 if the lepton polarization can be set to a value of $\lambda = v/a = 1 - 4\sin^2\theta \approx .13$. Similarly, the difference $\sigma^+ - \sigma^-$ reduces to xG_3 for $\lambda = 2va/(v^2+a^2) \approx 0.26$. Tuning the polarization is relatively easy for muon beams because λ is determined by the kinematics of the $\pi(K) \to \mu\nu$ decay [13]. Electron beams can be longitudinally polarized [14] although presently it is uncertain whether specific λ values can be prescribed.

iii) The analysis of cross sections within the parton model allows to determine quark distribution functions and to derive new QPM relations. In this context, the use of isoscalar targets is very important because the number of structure functions reduces to the two independent combinations $\sum(q+\bar{q})$ and $\sum(q-\bar{q})$.

Summarizing, new insight into the hadron structure can be expected, in particular at collider (ep) energies where no competing neutrino beam will exist and the inverse charged current reaction $ep \to \nu_e X$ is certainly measurable less accurately than $ep \to eX$.

Appendix: General Discussion of Cross-Section Combinations

Out of $\sigma^+(\lambda_1)$ and $\sigma^-(\lambda_2)$, (2.3–5), one can construct six independent combinations, i.e.

$$A^\pm = \tfrac{1}{2}[\sigma^\pm(\lambda_1) - \sigma^\pm(\lambda_2)]$$
$$S^\pm = \tfrac{1}{2}[\sigma^\pm(\lambda_1) + \sigma^\pm(\lambda_2)]$$
$$B_\pm = \tfrac{1}{2}[\sigma^+(\lambda_1) \pm \sigma^-(\lambda_2)]$$

Based on (2–4) and using

$$\sigma = \frac{d^2\sigma}{dQ^2 dx}\frac{Q^4 x}{2\pi\alpha^2}$$

they read as

$$A^\pm = L_-\cdot\{Y_-[v\cdot\kappa xG_3 - (v^2+a^2)\kappa^2\cdot xH_3] \pm Y_+[-a\kappa G_2 + 2av\cdot\kappa^2\cdot H_2]\}$$

$$S^\pm = Y_+\cdot[F_2 + (-v \mp aL_+)\cdot\kappa G_2 + (v^2+a^2 \pm 2avL_+)\kappa^2 xH_2] + Y_-\cdot[(vL_+ \pm a)\cdot\kappa xG_3 + (-L_+(v^2+a^2) \mp 2av)\kappa^2 xH_3]$$

$$B_+ = Y_+[F_2 - (v + aL_-)\kappa G_2 + (v^2+a^2+2avL_-)\kappa^2 H_2] + L_+ Y_-[v\kappa xG_3 - (v^2+a^2)\kappa^2 xH_3]$$

$$B_- = L_+ Y_+[-a\kappa G_2 + 2av\kappa^2 H_2] + Y_-[(a + L_- v)\kappa xG_3 - (2av + L_-(v^2+a^2))\kappa^2 xH_3]$$

with $L_\pm = (\lambda_1 \pm \lambda_2)/2$.

The parity violating combinations $A^\pm$ are scaled by L_- and contain four different structure functions. Thus they offer no simple approach to the study of structure functions. The combinations $S^\pm$ depend on L_+ and contain the full set of structure functions. Their complexity may be reduced a little bit by special choices of L_+ so that one of the coefficients vanishes. Nevertheless, a mixture of four structure functions remains. Contrary to $A^\pm$ and $S^\pm$, the beam conjugation combinations $B_\pm$ show a much more interesting behaviour:

Firstly it appears to be possible to eliminate two structure functions of the same type at once by the choice $L_+ = 0$, i.e. $\lambda_1 = -\lambda_2$. This leaves only structure functions of the other type in $B_\pm$. Furthermore, the polarization difference L_- remains arbitrary and can be used to remove either G_2 from B_+ or xH_3 from B_- (see Sect. 4).

Of course, all structure functions can be determined separately if one allows for the combination of more than two cross sections. For experimental reasons this seems to be excluded.

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Note added in proof. Regarding the prospects to measure $xG_3(x, Q^2)$ at an ep collider like HERA one should notice the need to run at modest energies with maximum luminosity since the xG_3 term (2.4) and (4.5) can only be large if $y = Q^2/sx$ is not too small.