WEAK NEUTRAL-CURRENT COUPLINGS OF MUONS

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Inversion of both the charge and the helicity of muon beams is considered as a possibility to determine the weak neutral-current couplings of muons, in particular the right-handed weak charge $I_3^R(\mu)$ and $\sin^2\theta$ without using the parton model.

Due to recently performed sensitive experiments, the weak neutral-current interaction of neutrinos, valence quarks and electrons is almost understood [1]. Different neutrino data have determined the vector and axial-vector couplings of u and d quarks [2]. The SLAC eD [3] and the Novosibirsk Bi experiments [4] have resolved the V – A ambiguity of elastic neutrino–electron scattering, thereby measuring the electron couplings [5,6]. Nothing is known, however, about the weak neutral-current couplings of muons. In this note, the possibility is considered to extract these couplings from deep inelastic polarized muon scattering at momentum transfers $Q^2 = O(100\ (\text{GeV}/c)^2)$. The muon couplings are of fundamental interest for neutral-current μe universality, for the single Z-boson hypothesis [7], for the existence of right-handed currents and of muon-induced parity violation. The vector coupling, if interpreted, e.g., in the Weinberg–Salam theory (WS), fixes the mixing angle $\sin^2\theta$. Three relations for $\sin^2\theta$ are derived, two of them without using the quark–parton model (QPM).

In deep inelastic muon scattering neutral currents are expected to be of the order of $k = Q^2G/\sqrt{2}\,2\pi\alpha = 1.79 \times 10^{-4}\, Q^2$ (in $(\text{GeV}/c)^2$) resulting from the interference of one-photon exchange with Z-boson exchange. Muons couple to the Z field by

$$g_Z \cdot \bar{\mu}\gamma^m(v_\mu - a_\mu\gamma_5)\mu \cdot Z_m \qquad (1)$$

with strength $g_Z^2/M_Z^2 = 2G/\sqrt{2}$. In SU(2) × U(1) gauge theories the couplings are

$$v_\mu = I_3^L + I_3^R + 2\sin^2\theta, \quad a_\mu = I_3^L - I_3^R, \qquad (2)$$

$I_3^{L(R)}$ being the left-handed (right-handed) weak μ^- charges. Neglecting radiative electromagnetic and weak corrections we can calculate the deep inelastic cross section of scattering muons, $d\sigma^\pm(\lambda)$, with charge ± and helicity λ off nucleons. Denoting the one-photon contribution by $d\sigma_0$ one gets [8,9]

$$d\sigma^\pm/d\sigma_0 = 1 - k[v_\mu V \pm a_\mu A + \lambda(\pm a_\mu V + v_\mu A)]\,. \qquad (3)$$

Here $V(x, Q^2)$ and $A(x, Q^2) = A_0(x, Q^2)g(y)$ are ratios of interference to electromagnetic structure functions depending on the dynamics and on the structure of the hadronic neutral current with $g(y) = (1-(1-y)^2)/(1+(1-y)^2)$ and V and A_0 defined as in ref. [9].

For a given magnitude of beam helicity λ there exist three independent cross section asymmetries: two parity violation asymmetries of the type measured at SLAC [3] and Serpukhov [10]:

$$A^\pm = \frac{d\sigma^\pm(+\lambda) - d\sigma^\pm(-\lambda)}{d\sigma^\pm(+\lambda) + d\sigma^\pm(-\lambda)} = -k\lambda(\pm a_\mu V + v_\mu A), \qquad (4)$$

and a third asymmetry to be measured by conjugation of the muon beam:

$$B = \frac{d\sigma^+(-\lambda) - d\sigma^-(+\lambda)}{d\sigma^+(-\lambda) + d\sigma^-(+\lambda)} = k(\lambda v_\mu - a_\mu)A. \qquad (5)$$

The measurement of these asymmetries is an obvious challenge for CERN SPS muon experiments reaching large Q^2 with high statistical accuracy [‡1].

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Below we concentrate on the beam conjugation asymmetry B because (i) the statistical accuracy of $B(\lambda)$ for positive λ is particularly high since it requires the use of only the high intensity forward part of the $\pi(K) \to \mu\nu$ decay spectrum; (ii) the only requirement for extracting a_μ and v_μ from B would be to control the axial-vector part of the hadronic current which is independent of $\sin^2\theta$; (iii) available neutral-current data predict the largest effects just for B [6]. According to ref. [9] one estimates for $\sin^2\theta = 1/4$ in the WS theory $V = 4/5, A = -9/5 \cdot g(y), a_\mu = -1/2$ and $v_\mu = 0$ giving at $Q^2 = 200\ (\mathrm{GeV}/c)^2$: $B(\lambda) = -3.2\, g(y)\%$, independently of λ, and $A^\pm = \pm 1.4\, \lambda\%$, independently of y.

The measurement of $B(\lambda)$ at two different helicities ‡2 is complete in the sense that it fixes the muon couplings. The vector coupling appears to be the slope of

$$B(\lambda)/(-kA) = a_\mu - \lambda v_\mu = -2(I_3^{\mathrm{R}} + \sin^2\theta), \quad \lambda = +1,$$
$$= +2(I_3^{\mathrm{L}} + \sin^2\theta), \quad \lambda = -1, \tag{6}$$

whereas the axial coupling is the intercept at $\lambda = 0$. Eq. (6) makes clear that the experimentally preferred helicity $\lambda \approx 1$ implies sensitivity of B to the right-handed muon coupling and to $\sin^2\theta$. For illustration $a_\mu - \lambda v_\mu$ versus λ is given in fig. 1 for standard $I_3^{\mathrm{L}} = -1/2$, keeping I_3^{R} as a free parameter. Four different assignments of I_3^{R} are considered $(-1, -1/2, 0, +1/2)$ corresponding to the right-handed multiplets [12,13]

$$\begin{pmatrix} \mathrm{M}^+ \\ \mathrm{M}^0 \\ \mu^- \end{pmatrix}_{\mathrm{R}}, \quad \begin{pmatrix} \mathrm{M}^0 \\ \mu^- \end{pmatrix}_{\mathrm{R}}, \quad \mu_{\mathrm{R}}^-, \quad \begin{pmatrix} \mu^- \\ \mathrm{M}^{--} \end{pmatrix}_{\mathrm{R}}, \tag{7}$$

containing heavy leptons M. The solid (dashed) curves in fig. 1 belong to $\sin^2\theta = 0.2\ (0.3)$. It is of importance

‡1 Note that all definitions and subsequent arguments are not only applicable to deep inelastic but also to elastic scattering if the ratios of structure functions V and A_0 are replaced by ratios of form factors.

‡2 Charge conjugation maintains the beam charge dependent part of the radiative corrections. The resulting electromagnetic asymmetry B_{elm} has been calculated to be positive and smaller than 1% below $Q^2/s = 0.5$ [11]. One gets rid of B_{elm} by subtracting B asymmetries at two different energies $E_1 < E_2$ since B_{elm} is very likely to be scale invariant. This subtraction at fixed (x, y) decreases the weak asymmetry by a factor $1 - E_1/E_2$.

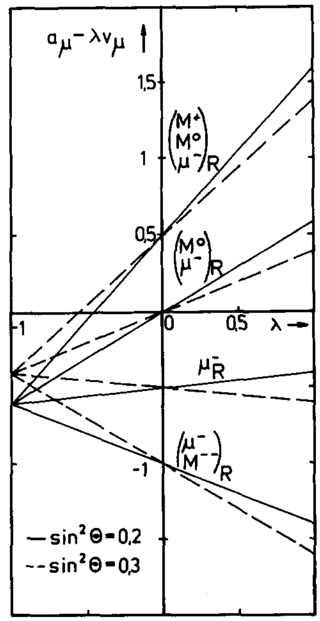

Fig. 1. $a_\mu - \lambda v_\mu$, eq. (6), as a function of the μ^- beam helicity λ for $I_3^{\mathrm{L}} = -1/2$ and $I_3^{\mathrm{R}} = (-1, -1/2, 0, +1/2)$. Solid (dashed) curves belong to $\sin^2\theta = 0.2\ (0.3)$.

that the variations at λ near 1 due to I_3^{R} are dominant as compared to what is expected from $\sin^2\theta$. Thus, using very forward produced muons, i.e., the standard SPS muon beam, the right-handed weak charge can be measured. Heavy leptons of a few GeV mass may also give directly detectable signals [14].

From the present status [1] the WS theory would be expected to be confirmed $(I_3^{\mathrm{L}} = -1/2, I_3^{\mathrm{R}} = 0)$. Then the next question would concern the details of this theory, i.e. the mixing angle and the Higgs multiplet structure which affects the ratio [1] $\rho = M_{\mathrm{W}}^2/M_{\mathrm{Z}}^2 \times \cos^2\theta$. The asymmetry $B(\lambda)$ at $\lambda = 0$ is independent of $\sin^2\theta$ (eq. (5)). Thus ρ is fixed by

$$\rho = M_{\mathrm{W}}^2/M_{\mathrm{Z}}^2 \cos^2\theta = 2B(0)/kA. \tag{8}$$

Neutral-current neutrino data giving $\rho = 0.98 \pm 0.05$ indicate a minimal Higgs structure [2]. Eq. (5) can be rewritten to determine $\sin^2\theta$ from the leptonic current as

$$\sin^2\theta = 1/4 + [2B(\lambda)/kA - 1]/4\lambda. \tag{9}$$

A similar relation using both parity violation asymmetries (eq. (4)) has been derived in ref. [15]. One can avoid the QPM calculation of A measuring $B(\lambda)$ at two different helicities. Calculating $B_1 = B(\lambda_1)$ and B_2

$= B(\lambda_2)$ at the same (Q^2, x) one gets independently of ρ

$$\sin^2\theta = 1/4 + [(B_2 - B_1)/(B_1\lambda_2 - B_2\lambda_1)]/4. \quad (10)$$

This relation expresses $\sin^2\theta$ in terms of measurable quantities only and is free of any dynamical assumption.

Recently it has been shown by several authors that the hadronic axial-vector current can be related by isospin invariance to the difference between antineutrino and neutrino charged-current cross sections [15–17]. This allows one to introduce a neutrino beam conjugation asymmetry, B_ν, being completely analogous to B (eq. (5)):

$$B_\nu = (\mathrm{d}\sigma^{\bar\nu} - \mathrm{d}\sigma^{\nu})/(\mathrm{d}\sigma^{\bar\nu} + \mathrm{d}\sigma^{\nu}). \quad (11)$$

B_ν is approximately [16] equal to $A \cdot 5/9$, giving

$$B(\lambda) = k(\lambda v_\mu - a_\mu)B_\nu \cdot 9/5. \quad (12)$$

Therefore, the muon couplings and the parameters of the WS theory are given by combining deep inelastic muon and neutrino scattering data at the same (Q^2, x). A third possibility to calculate $\sin^2\theta$ is then:

$$\sin^2\theta = 1/4 + [10B(\lambda)/9kB_\nu - 1]/4\lambda. \quad (13)$$

The present world average for $\sin^2\theta$ is 0.23 ± 0.02 [1]. Thus almost equal beam conjugations are expected in muon and neutrino scattering which differ only by the corresponding coupling constants and propagators, respectively:

$$B \cdot 2\pi\alpha/Q^2 \approx B_\nu \cdot G/\sqrt{2}. \quad (14)$$

A fundamental problem to be investigated with charged lepton beams is parity violation. The natural way to search for parity violation would be to measure the asymmetries $A^\pm$ (eq. (4)) containing only V – A combinations. Nevertheless, one can ask how to study parity violation when measuring B. The answer is obvious after rewriting B for different $\mu^\pm$ helicities as

$$B(\lambda_1, \lambda_2) = (\mathrm{d}\sigma^+(\lambda_1) - \mathrm{d}\sigma^-(\lambda_2))/(\mathrm{d}\sigma^+(\lambda_1) + \mathrm{d}\sigma^-(\lambda_2))$$
$$= -k[a_\mu A + v_\mu A \cdot (\lambda_1 - \lambda_2)/2 - a_\mu V \cdot (\lambda_1 + \lambda_2)/2]. \quad (15)$$

For large $\lambda_1 - \lambda_2$, as considered above, the measurement is sensitive to $v_\mu A$. For electrons, this combination is suppressed in the heavy atom experiments. In the WS theory it is expected to be small. For large $\lambda_1 + \lambda_2$ the measurement is sensitive to $a_\mu V$. This combination has been essentially observed at SLAC and Novosibirsk. In the WS theory at $\sin^2\theta = 1/4$ one estimates $a_\mu V = -0.4$ to be compared with the parity conserving contribution to B, $a_\mu A = 0.9g(y)$. Note that only $a_\mu V$ should survive if $B(\lambda_1, \lambda_2)$ is calculated for y tending to zero.

To summarize, the muon beam conjugation asymmetry B, eq. (5), is of particular interest since it is measurable rather accurately and promises to determine the weak neutral-current couplings of muons. Helicity λ near 1 (forward produced muons) implies particular sensitivity of B to the right-handed weak charge $I_3^R(\mu)$. Several relations for $\sin^2\theta$ have been derived which are based either on the parton model or, independently of it, on two measurements of $B(\lambda)$ and on a neutrino beam conjugation asymmetry, respectively. The helicity and y dependence of B give insight into the question of muon-induced parity violation in a new range of momentum transfers.

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