Flavour-Covariant Rate Equations for Resonant Leptogenesis

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Based on:

- P. S. B. Dev, P. M., A. Pilaftsis and D. Teresi,
  Flavour Covariant Rate Equations:
  an Application to Resonant Leptogenesis,
  Nucl. Phys. B886 (2014) 569-664, arXiv:1404.1003

- P. S. B. Dev, P. M., A. Pilaftsis and D. Teresi,
  Flavour Covariant Formalism for Resonant Leptogenesis,
  in proc. ICHEP2014, arXiv:1409.8263

- P. S. B. Dev, P. M., A. Pilaftsis and D. Teresi,
  Kadanoff-Baym Approach to
  Flavour Mixing and Oscillations in Resonant Leptogenesis,
  arXiv:1410.6434 (to appear in Nucl. Phys. B)

- (P. Millington and A. Pilaftsis, Phys. Rev. D88 (2013) 8, 085009,
  arXiv:1211.3152; Phys. Lett. B724 (2013) 56, arXiv:1304.7249)

[Please also see the comprehensive lists of references in the above.]

Outline

- Introduction and Motivation
- Scenarios of Leptogenesis
- Flavour-Covariant Formulation
- Semi-Classical Transport Phenomena
- Numerical Results
- Field-Theoretic Transport Phenomena
- Conclusions

Experimental Observations

- Cosmology: baryonic matter dominates over anti-baryonic matter in the Universe
  [Planck temperature maps with WMAP polarizationd data]

$$\eta_B = \frac{n_b - n_{\bar{b}}}{n_\gamma} = (6.04 \pm 0.08) \times 10^{-10}$$

- Particle physics: the light-neutrino mass scale (best fit values)
  [Global analysis from F. Capozzi et al., PRD89 (2014) 093018]

$$\Delta m^2_{\rm sol} = 7.54 \times 10^{-5}\ {\rm eV}^2$$
$$\Delta m^2_{\rm atm} = 2.44 \times 10^{-3}\ {\rm eV}^2$$

Baryon Asymmetry of the Universe (BAU)

In order to generate the observed baryon asymmetry, we must satisfy the 3 **Sakharov conditions**:
[A. D. Sakharov, JETP Lett. 5 (1967) 24]

1. C and CP violation
2. B violation
3. departure from thermodynamic equilibrium

Leptogenesis in a Nut-Shell

- Before the electroweak phase transition, out-of-equilibrium (Sakharov # 3) *CP*-violating (Sakharov # 1) heavy Majorana neutrino decays produce a net lepton number L.

$$\Gamma(X \to Y) \; < \; H \; = \; \left(\frac{4\pi^3}{45}\right)^{1/2} g_*^{1/2} \frac{T^2}{M_{\rm Pl}} \qquad t \; = \; \frac{M_{\rm Pl}}{34\, T^2}$$

- During the electroweak phase transition, equilibriated $B+L$-violating (Sakharov # 2) Sphaleron decays reprocess the remaining lepton number L into a net baryon number B.

Leptogenesis Scenarios (Some)

- Thermal leptogenesis:
  heavy-neutrino masses of order $M_{\mathrm{GUT}} \sim 10^{16}$ GeV.
  [M. Fukugita, T. Yanagida, PLB174 (1986) 45]

- 'Vanilla' leptogenesis:
  hierarchical heavy-neutrino masses $m_{N_1} \ll m_{N_2} < m_{N_3}$.
  - solar and atmospheric ν oscillation data: $m_{N_1} \gtrsim 10^9$ GeV.
    [S. Davidson, A. Ibarra, PLB535 (2002) 25;
    W. Buchmuller, P. Di Bari, M. Plümacher, NPB643 (2002) 367]
  - potential conflict with gravitino overproduction bound
    $T_{\mathrm{R}} \lesssim 10^6$ - 10^9 GeV (supergravity embedding).
    [see e.g., M. Kawasaki, K. Kohri, T. Moroi, A. Yotsuyanagi, PRD78 (2008) 065011]

Resonant Leptogenesis

[See A. Pilaftsis, NPB504 (1997) 61; PRD56 (1997) 5431; A. Pilaftsis, T. Underwood, NPB692 (2004) 303]

- Quasi-degenerate mass spectrum $\Delta m_N \sim \Gamma_{N_{1,2}} \ll m_{N_{1,2}}$.
- Heavy-neutrino self-energy effects on the leptonic CP-asymmetry are resonantly enhanced.
  [See also E. A. Paschos, U. Türke, PR178 (1989) 145 (Kaon sector); V. A. Kuzmin, V. A. Rubakov, M. E. Shaposhnikov PLB155 (1985) 36; J. Liu, G. Segrè, PRD 48 (1993) 4609; M. Flanz, E. Paschos, U. Sarkar, PLB345 (1995) 248; L. Covi, E. Roulet, F. Vissani, PLB384 (1996) 169, ...]
- Heavy-neutrino masses at order the **electroweak scale**.
- Variant: **Resonant ℓ-Genesis (RL$_\ell$)**
  [A. Pilaftsis, PRL 95 , 081602 (2005); A. Pilaftsis, T. Underwood, PRD72 (2005) 113001; F. F. Deppisch, A. Pilaftsis, PRD 83, 076007 (2011)]
  - Sphaleron processes preserve $X_i = B/3 - L_i$.
    [J. A. Harvey, M. S. Turner, PRD42 (1990) 3344; H. Dreiner, G. G. Ross, NPB410 (1993) 188; J. M. Cline, K. Kainulainen, K. A. Olive, PRD49 (1994) 6394]
  - Baryon asymmetry protected from Sphaleron washout if a single flavour ℓ remains out of equilibrium.
  - Electroweak scale heavy-neutrino masses with **sizable couplings** to other SM fermion flavours.

Transport Phenomena

We want to describe the **evolution** of **particle number densities**.

- **Semi-classical Boltzmann equations**
  [see e.g. E. W. Kolb, S. Wolfram, NPB172 (1980) 224;
  for the so-called 'density matrix' formalism, see e.g. G. Sigl, G. Raffelt, NPB406 (1993) 423]
  - Pros: more intuitive, unambiguous definition of physical observables
  - Cons: quantum effects must be included effectively
- **Non-equilibrium QFT/Kadanoff-Baym equations**
  [L. Kadanoff and G. Baym, *Quantum Statistical Mechanics*, Benjamin, New York (1962); for an extensive list of other works (from *many* authors) see B. S. P. Dev et al., NPB886 (2014) 569–664; arXiv:1410.6434]
  - Pros: quantum effects included from the outset
  - Cons: less intuitive, potential ambiguity in extracting physical observables

Whichever approach we use, we require a **fully flavour-covariant formalism** in order to capture the flavour effects pertinent to RL.
[. . . ; Akhmedov, Rubakov, Smirnov, PRL81 (1998); Barbieri, Creminelli, Strumia, Tetradis, NPB575 (2000); Endoh, Morozumi, Xiong, PTP111 (2004); Pilaftsis, PRL95 (2005); Pilaftsis, Underwood, PRD72 (2005); Asaka, Shaposhnikov, PLB620 (2005); Di Bari, NPB727 (2005); Abada, Davidson, Josse-Michaux, Losada, Riotto, JCAP 0604 (2006); Nardi, Nir, Racker, Roulet, JHEP 0601 (2006) 164; Drewes, Garbrecht, JHEP 1303 (2013); . . .]

Flavour-Covariant Formulation

Flavour Rotations

$$-\mathcal{L}_N \;=\; h_l{}^{\alpha}\,\overline{L}^l\,\widetilde{\Phi}\,N_{\mathrm{R},\alpha} \;+\; \frac{1}{2}\,\overline{N}^C_{\mathrm{R},\alpha}[M_N]^{\alpha\beta}N_{\mathrm{R},\beta} \;+\; \mathrm{H.c.}$$

Unitary flavour transformations:

- **Charged-lepton** fields transform as vectors of $U(3)_L$, i.e. $L_l \;\rightarrow\; V_l{}^m L_m$ and $[L_l]^\dagger \;\equiv\; L^l \;\rightarrow\; V^l{}_m L^m$.
- **Heavy-neutrino** fields transform as vectors of $U(3)_N$, i.e. $N_{\mathrm{R},\alpha} \;\rightarrow\; U_\alpha{}^\beta N_{\mathrm{R},\beta}$ and $[N_{\mathrm{R},\alpha}]^\dagger \;\equiv\; N_{\mathrm{R}}^\alpha \;\rightarrow\; U^\alpha{}_\beta N_{\mathrm{R}}^\beta$.
- **Majorana mass** transforms as a tensor of $U(3)_N$, $[M_N]^{\alpha\beta} \;\rightarrow\; U^\alpha{}_\gamma U^\beta{}_\delta [M_N]^{\gamma\delta}$.
- Invariant Lagrangian if **heavy-neutrino Yukawa couplings** transform under $U(3)_L \,\times\, U(3)_N$ as $h_l{}^\alpha \;\rightarrow\; V_l{}^m U^\alpha{}_\beta h_m{}^\beta$.

Flavour-Covariant Formulation

Quantization

- Field operators:

$$L_l(x) \;=\; \sum_s \int_{\mathbf{p}} \Big[\big(2E_L(\mathbf{p})\big)^{-1/2}\Big]_l^{\;i} \Big(\big[e^{-ip\cdot x}\big]_i^{\;j} [u(\mathbf{p},s)]_j^{\;k}\, b_k(\mathbf{p},s,0) + \big[e^{ip\cdot x}\big]_i^{\;j} [v(\mathbf{p},s)]_j^{\;k}\, d_k^{\dagger}(\mathbf{p},s,0) \Big)$$

- Creation and annihilation operators:

$$\{b_k(\mathbf{p},s,\tilde{t}),\; b^l(\mathbf{p}',s',\tilde{t})\} \;=\; (2\pi)^3 \delta_k^{\;l} \delta^{(3)}(\mathbf{p}-\mathbf{p}')$$
$$\{d^{\dagger,l}(\mathbf{p},s,\tilde{t}),\; d_k^{\dagger}(\mathbf{p}',s',\tilde{t})\} \;=\; (2\pi)^3 \delta_k^{\;l} \delta^{(3)}(\mathbf{p}-\mathbf{p}')$$

- Particle and anti-particle creation operators b^l and $d_l^{\dagger}$ transform in **different** representations of $U(3)_L$.

Flavour-Covariant Formulation

Generalised Discrete Symmetry Transformations

- Charge-conjugation (C) and time-reversal (T) only have simple forms in the mass eigenbasis.
- In a **general flavour basis**, we must introduce generalised $\widetilde{C}$ and $\widetilde{T}$ transformations

$$[b_l(\mathbf{p}, s, \tilde{t})]^{\widetilde{C}} \equiv \mathcal{G}^{lm}[b_l(\mathbf{p}, s, \tilde{t})]^{C} = -\, i d^{l}(\mathbf{p}, s, \tilde{t})$$

$$[b_l(\mathbf{p}, s, \tilde{t})]^{\widetilde{T}} \equiv \mathcal{G}_{lm}[b_m(\mathbf{p}, s, \tilde{t})]^{T} = b_l(-\mathbf{p}, s, -\tilde{t})$$

with $\boldsymbol{\mathcal{G}} \equiv \boldsymbol{V}\boldsymbol{V}^{\mathsf{T}}$.

- Generalised **Majorana constraint**

$$d^{\dagger,\alpha}(\mathbf{k}, -r, \tilde{t}) \equiv G^{\alpha\beta} b_{\beta}(\mathbf{k}, r, \tilde{t})$$

with $\boldsymbol{G} \equiv \boldsymbol{U}\boldsymbol{U}^{\mathsf{T}}$

Flavour-Covariant Formulation

Particle Number Densities

- **Number densities** become **matrices** in **flavour space**

$$[n^L(\mathbf{p},t)]_l{}^m \;\equiv\; \mathcal{V}^{-1}\,\langle b^m(\mathbf{p},\tilde{t})b_l(\mathbf{p},\tilde{t})\rangle_t$$
$$[\overline{n}^L(\mathbf{p},t)]_l{}^m \;\equiv\; \mathcal{V}^{-1}\,\langle d_l^\dagger(\mathbf{p},\tilde{t})d^{\dagger,m}(\mathbf{p},\tilde{t})\rangle_t$$
$$[n^N(\mathbf{k},t)]_\alpha{}^\beta \;\equiv\; \mathcal{V}^{-1}\,\langle a^\beta(\mathbf{k},\tilde{t})a_\alpha(\mathbf{k},\tilde{t})\rangle_t$$

- $\widetilde{C}$ transformation properties:

$$[\boldsymbol{n}^L]^{\widetilde{C}} \;=\; [\overline{\boldsymbol{n}}^L]^{\mathsf{T}} \qquad [\boldsymbol{n}^N]^{\widetilde{C}} \;=\; [\overline{\boldsymbol{n}}^N]^{\mathsf{T}}$$

- Introduce $\widetilde{C}P$-“even” and -“odd” quantities:

$$\underline{\boldsymbol{n}}^N \;=\; \frac{1}{2}(\boldsymbol{n}^N + \overline{\boldsymbol{n}}^N)$$
$$\boldsymbol{\delta n}^N \;=\; \boldsymbol{n}^N - \overline{\boldsymbol{n}}^N\,, \qquad \boldsymbol{\delta n}^L \;=\; \boldsymbol{n}^L - \overline{\boldsymbol{n}}^L$$

- Number densities are of the form

$$\boldsymbol{n}^X(t) \;\equiv\; \langle \check{\boldsymbol{n}}^X(\tilde{t};\tilde{t}_i)\rangle_t \;=\; \mathrm{Tr}\Big\{\rho(\tilde{t};\tilde{t}_i)\,\check{\boldsymbol{n}}^X(\tilde{t};\tilde{t}_i)\Big\}$$

- Take the time derivative, use the von Neumann (for density operator ρ) and Heisenberg (for number operator $\check{\boldsymbol{n}}^X$) equations and make a Wigner-Weisskopf approximation to obtain the **Markovian master equation**

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{n}^X(\mathbf{k},t) \;\simeq\; \langle[H_0^X,\, \check{\boldsymbol{n}}^X(\mathbf{k},t)]\rangle_t \\ \quad -\; \frac{1}{2}\int_{-\infty}^{+\infty}\mathrm{d}t'\;\langle[H_{\mathrm{int}}(t'),\,[H_{\mathrm{int}}(t),\,\check{\boldsymbol{n}}^X(\mathbf{k},t)]]\rangle_t$$

Semi-Classical Transport Equations

Markovian Master Equation

$$\frac{\mathrm{d}}{\mathrm{d}t}[n^L_{s_1 s_2}(\mathbf{p},t)]_l{}^m = -i[E_L(\mathbf{p}),\ n^L_{s_1 s_2}(\mathbf{p},t)]_l{}^m + [C^L_{s_1 s_2}(\mathbf{p},t)]_l{}^m$$

$$\frac{\mathrm{d}}{\mathrm{d}t}[n^N_{r_1 r_2}(\mathbf{k},t)]_\alpha{}^\beta = -i[E_N(\mathbf{k}),\ n^N_{r_1 r_2}(\mathbf{k},t)]_\alpha{}^\beta + [C^N_{r_1 r_2}(\mathbf{k},t)]_\alpha{}^\beta + G_{\alpha\lambda}[\overline{C}^N_{r_2 r_1}(\mathbf{k},t)]_\mu{}^\lambda] G^{\mu\beta}$$

with **collision terms**

$$[C^L_{s_1 s_2}(\mathbf{p},t)]_l{}^m \supset -\frac{1}{2}[\mathcal{F}_{s_1 s r_1 r_2}(\mathbf{p},\mathbf{q},\mathbf{k},t)]_l{}^n{}_\alpha{}^\beta [\Gamma_{s s_2 r_1 r_2}(\mathbf{p},\mathbf{q},\mathbf{k},t)]_n{}^m{}_\beta{}^\alpha$$

- **statistical tensors**
  $\mathcal{F} = n^\Phi \boldsymbol{n}^L \otimes (\mathbf{1} - \boldsymbol{n}^N) - (1 + n^\Phi)(1 - \boldsymbol{n}^L) \otimes \boldsymbol{n}^N$
- **rank-4 absorptive rate tensors** Γ (here for $N \leftrightarrow L\Phi$)

Collision Rates

Generalised Optical Theorem

- **Optical theorem:**

$$S^\dagger S = SS^\dagger = \mathbb{I} \qquad S = \mathbb{I} + i\mathcal{T} \qquad 2\mathrm{Im}\mathcal{T} = \mathcal{T}^\dagger \mathcal{T}$$

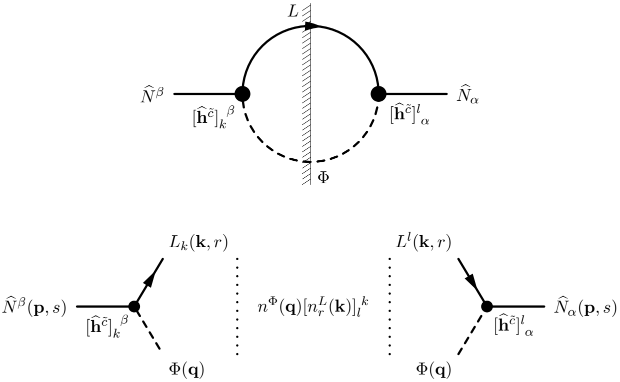
- Completeness of the **Fock space** is a flavour, spin, isospin **singlet** and a Lorentz **scalar**.
- Non-trivial structure arises from **density operator** ρ:

$$\langle 2\mathrm{Im}\mathcal{T}\rangle = \langle \mathcal{T}^\dagger \mathcal{T}\rangle = \mathrm{Tr}\,\mathcal{T}\rho\mathcal{T}^\dagger$$

$$\langle 2\mathrm{Im}\mathcal{T}\rangle = \sum_{A,B,C} \langle A|\mathcal{T}|B\rangle \langle B|\rho|C\rangle \langle C|\mathcal{T}^\dagger|A\rangle$$

Collision Rates

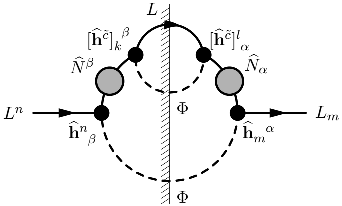
Decays and Inverse Decays

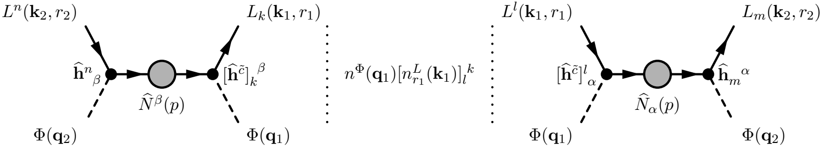


Cuts of partial self-energies give **rank-4 rates** $[\gamma(L\Phi \to N)]_k{}^l{}_{\alpha}{}^{\beta}$.

Collision Rates

Scatterings





Cuts of partial self-energies give **rank-4 rates** $[\gamma(L\Phi \to L\Phi)]_k{}^l{}_m{}^n$.

Application to Resonant Leptogenesis

- Classical statistics: $1 \pm n^X \simeq 1$
- Kinetic equilibrium and average heavy-neutrino mass: $m_N^2 \equiv \mathcal{N}_N^{-1} \mathrm{Tr}\, \boldsymbol{M}_N^\dagger \boldsymbol{M}_N$
- Massless charged-leptons (single helicity state populated)
- Equally-populated heavy-neutrino helicity states
- Small departure from equilibrium: $[n^L]_l{}^m + [\overline{n}^L]_l{}^m \simeq 2 n_{\mathrm{eq}}^L \delta_l{}^m$
- Heavy-neutrino mixing by replacing tree-level Yukawa couplings $h_l{}^\alpha$ by resummed Yukawa couplings $\mathbf{h}_l^\alpha$ and $[\mathbf{h}^{\tilde{c}}]_l{}^\alpha$.

$$[\gamma_{L\Phi}^N]_l{}^m{}_\alpha{}^\beta \;\propto\; \mathbf{h}^m{}_\alpha \mathbf{h}_l{}^\beta + [\mathbf{h}^{\tilde{c}}]^m{}_\alpha [\mathbf{h}^{\tilde{c}}]_l{}^\beta$$

$$[\delta\gamma_{L\Phi}^N]_l{}^m{}_\alpha{}^\beta \;\propto\; \mathbf{h}^m{}_\alpha \mathbf{h}_l{}^\beta - [\mathbf{h}^{\tilde{c}}]^m{}_\alpha [\mathbf{h}^{\tilde{c}}]_l{}^\beta$$

- Thermal RIS subtraction to avoid double-counting
- Charged-lepton decoherence interactions (involving right-handed charged-leptons)

Application to Resonant Leptogenesis

Final Rate Equations

$$\frac{H_N n^\gamma}{z} \frac{\mathrm{d}[\underline{\eta}^N]_\alpha{}^\beta}{\mathrm{d}z} = -\, i \frac{n^\gamma}{2} \Big[\mathcal{E}_N,\, \delta\eta^N\Big]_\alpha{}^\beta + \Big[\widetilde{\mathrm{Re}}\big(\gamma^N_{L\Phi}\big)\Big]_\alpha{}^\beta - \frac{1}{2\eta^N_{\mathrm{eq}}}\Big\{\underline{\eta}^N,\, \widetilde{\mathrm{Re}}\big(\gamma^N_{L\Phi}\big)\Big\}_\alpha{}^\beta$$

$$\frac{H_N n^\gamma}{z} \frac{\mathrm{d}[\delta\eta^N]_\alpha{}^\beta}{\mathrm{d}z} = -\, 2i\, n^\gamma \Big[\mathcal{E}_N,\, \underline{\eta}^N\Big]_\alpha{}^\beta + 2i\Big[\widetilde{\mathrm{Im}}\big(\delta\gamma^N_{L\Phi}\big)\Big]_\alpha{}^\beta - \frac{i}{\eta^N_{\mathrm{eq}}}\Big\{\underline{\eta}^N,\, \widetilde{\mathrm{Im}}\big(\delta\gamma^N_{L\Phi}\big)\Big\}_\alpha{}^\beta - \frac{1}{2\eta^N_{\mathrm{eq}}}\Big\{\delta\eta^N,\, \widetilde{\mathrm{Re}}\big(\gamma^N_{L\Phi}\big)\Big\}_\alpha{}^\beta$$

$$\frac{H_N n^\gamma}{z} \frac{\mathrm{d}[\delta\eta^L]_l{}^m}{\mathrm{d}z} = -\, [\delta\gamma^N_{L\Phi}]_l{}^m + \frac{[\underline{\eta}^N]_\beta{}^\alpha}{\eta^N_{\mathrm{eq}}}[\delta\gamma^N_{L\Phi}]_l{}^m{}_\alpha{}^\beta + \frac{[\delta\eta^N]_\beta{}^\alpha}{2\eta^N_{\mathrm{eq}}}[\gamma^N_{L\Phi}]_l{}^m{}_\alpha{}^\beta - \frac{1}{3}\Big\{\delta\eta^L,\, \gamma^{L\Phi}_{L^{\tilde{c}}\Phi^{\tilde{c}}} + \gamma^{L\Phi}_{L\Phi}\Big\}_l{}^m - \frac{2}{3}[\delta\eta^L]_k{}^n\Big([\gamma^{L\Phi}_{L^{\tilde{c}}\Phi^{\tilde{c}}} - [\gamma^{L\Phi}_{L\Phi}]\Big)_n{}^k{}_l{}^m - \frac{2}{3}\Big\{\delta\eta^L,\, \gamma_{\mathrm{dec}}\Big\}_l{}^m + [\delta\gamma^{\mathrm{back}}_{\mathrm{dec}}]_l{}^m$$

Application to Resonant Leptogenesis

Final Rate Equations: Mixing

$$\frac{H_N n^\gamma}{z}\frac{\mathrm{d}[\underline{\eta}^N]_\alpha{}^\beta}{\mathrm{d}z} = -\, i\,\frac{n^\gamma}{2}\Big[\mathcal{E}_N,\ \delta\eta^N\Big]_\alpha{}^\beta + \Big[\widetilde{\mathrm{Re}}\big(\gamma^N_{L\Phi}\big)\Big]_\alpha{}^\beta - \frac{1}{2\eta^N_{\mathrm{eq}}}\Big\{\underline{\eta}^N,\ \widetilde{\mathrm{Re}}\big(\gamma^N_{L\Phi}\big)\Big\}_\alpha{}^\beta$$

$$\frac{H_N n^\gamma}{z}\frac{\mathrm{d}[\delta\eta^N]_\alpha{}^\beta}{\mathrm{d}z} = -\, 2i\, n^\gamma\Big[\mathcal{E}_N,\ \underline{\eta}^N\Big]_\alpha{}^\beta + 2i\Big[\widetilde{\mathrm{Im}}\big(\delta\gamma^N_{L\Phi}\big)\Big]_\alpha{}^\beta - \frac{i}{\eta^N_{\mathrm{eq}}}\Big\{\underline{\eta}^N,\ \widetilde{\mathrm{Im}}\big(\delta\gamma^N_{L\Phi}\big)\Big\}_\alpha{}^\beta - \frac{1}{2\eta^N_{\mathrm{eq}}}\Big\{\delta\eta^N,\ \widetilde{\mathrm{Re}}\big(\gamma^N_{L\Phi}\big)\Big\}_\alpha{}^\beta$$

$$\frac{H_N n^\gamma}{z}\frac{\mathrm{d}[\delta\eta^L]_l{}^m}{\mathrm{d}z} = -\,[\delta\gamma^N_{L\Phi}]_l{}^m + \frac{[\underline{\eta}^N]_\beta{}^\alpha}{\eta^N_{\mathrm{eq}}}[\delta\gamma^N_{L\Phi}]_l{}^m{}_\alpha{}^\beta + \frac{[\delta\eta^N]_\beta{}^\alpha}{2\eta^N_{\mathrm{eq}}}[\gamma^N_{L\Phi}]_l{}^m{}_\alpha{}^\beta - \frac{1}{3}\Big\{\delta\eta^L,\ \gamma^{L\Phi}_{\tilde{L}^c\Phi\tilde{c}} + \gamma^{L\Phi}_{L\Phi}\Big\}_l{}^m - \frac{2}{3}[\delta\eta^L]_k{}^n\Big([\gamma^{L\Phi}_{\tilde{L}^c\Phi\tilde{c}} - [\gamma^{L\Phi}_{L\Phi}]\Big)_n{}^k{}_l{}^m - \frac{2}{3}\Big\{\delta\eta^L,\ \gamma_{\mathrm{dec}}\Big\}_l{}^m + [\delta\gamma^{\mathrm{back}}_{\mathrm{dec}}]_l{}^m$$

Application to Resonant Leptogenesis

Final Rate Equations: Oscillations

$$\frac{H_N n^\gamma}{z} \frac{\mathrm{d}[\underline{\eta}^N]_\alpha^{\ \beta}}{\mathrm{d}z} = -\, i\frac{n^\gamma}{2}\Big[\mathcal{E}_N,\, \delta\eta^N\Big]_\alpha^{\ \beta} + \Big[\widetilde{\mathrm{Re}}\big(\gamma^N_{L\Phi}\big)\Big]_\alpha^{\ \beta} - \frac{1}{2\eta^N_{\mathrm{eq}}}\Big\{\underline{\eta}^N,\, \widetilde{\mathrm{Re}}\big(\gamma^N_{L\Phi}\big)\Big\}_\alpha^{\ \beta}$$

$$\frac{H_N n^\gamma}{z} \frac{\mathrm{d}[\delta\eta^N]_\alpha^{\ \beta}}{\mathrm{d}z} = -\, 2i\, n^\gamma\Big[\mathcal{E}_N,\, \underline{\eta}^N\Big]_\alpha^{\ \beta} + 2i\Big[\widetilde{\mathrm{Im}}\big(\delta\gamma^N_{L\Phi}\big)\Big]_\alpha^{\ \beta} - \frac{i}{\eta^N_{\mathrm{eq}}}\Big\{\underline{\eta}^N,\, \widetilde{\mathrm{Im}}\big(\delta\gamma^N_{L\Phi}\big)\Big\}_\alpha^{\ \beta}$$
$$-\, \frac{1}{2\eta^N_{\mathrm{eq}}}\Big\{\delta\eta^N,\, \widetilde{\mathrm{Re}}\big(\gamma^N_{L\Phi}\big)\Big\}_\alpha^{\ \beta}$$

$$\frac{H_N n^\gamma}{z} \frac{\mathrm{d}[\delta\eta^L]_l^{\ m}}{\mathrm{d}z} = -\, [\delta\gamma^N_{L\Phi}]_l^{\ m} + \frac{[\underline{\eta}^N]_\beta^{\ \alpha}}{\eta^N_{\mathrm{eq}}}[\delta\gamma^N_{L\Phi}]_l^{\ m}{}_\alpha^{\ \beta} + \frac{[\delta\eta^N]_\beta^{\ \alpha}}{2\eta^N_{\mathrm{eq}}}[\gamma^N_{L\Phi}]_l^{\ m}{}_\alpha^{\ \beta}$$
$$-\, \frac{1}{3}\Big\{\delta\eta^L,\, \gamma^{L\Phi}_{L^{\tilde{c}}\Phi^{\tilde{c}}} + \gamma^{L\Phi}_{L\Phi}\Big\}_l^{\ m} - \frac{2}{3}[\delta\eta^L]_k^{\ n}\Big([\gamma^{L\Phi}_{L^{\tilde{c}}\Phi^{\tilde{c}}} - [\gamma^{L\Phi}_{L\Phi}]\Big)_{n\ \ l}^{\ \ k\ m}$$
$$-\, \frac{2}{3}\Big\{\delta\eta^L,\, \gamma_{\mathrm{dec}}\Big\}_l^{\ m} + [\delta\gamma^{\mathrm{back}}_{\mathrm{dec}}]_l^{\ m}$$

Application to Resonant Leptogenesis

Final Rate Equations: Decoherence

$$\frac{H_N n^\gamma}{z}\frac{\mathrm{d}[\underline{\eta}^N]_\alpha{}^\beta}{\mathrm{d}z} = -\, i\,\frac{n^\gamma}{2}\Big[\mathcal{E}_N,\, \delta\eta^N\Big]_\alpha{}^\beta + \Big[\widetilde{\mathrm{Re}}\big(\gamma^N_{L\Phi}\big)\Big]_\alpha{}^\beta - \frac{1}{2\eta^N_{\mathrm{eq}}}\Big\{\underline{\eta}^N,\, \widetilde{\mathrm{Re}}\big(\gamma^N_{L\Phi}\big)\Big\}_\alpha{}^\beta$$

$$\frac{H_N n^\gamma}{z}\frac{\mathrm{d}[\delta\eta^N]_\alpha{}^\beta}{\mathrm{d}z} = -\, 2i\, n^\gamma\Big[\mathcal{E}_N,\, \underline{\eta}^N\Big]_\alpha{}^\beta + 2i\Big[\widetilde{\mathrm{Im}}\big(\delta\gamma^N_{L\Phi}\big)\Big]_\alpha{}^\beta - \frac{i}{\eta^N_{\mathrm{eq}}}\Big\{\underline{\eta}^N,\, \widetilde{\mathrm{Im}}\big(\delta\gamma^N_{L\Phi}\big)\Big\}_\alpha{}^\beta - \frac{1}{2\eta^N_{\mathrm{eq}}}\Big\{\delta\eta^N,\, \widetilde{\mathrm{Re}}\big(\gamma^N_{L\Phi}\big)\Big\}_\alpha{}^\beta$$

$$\frac{H_N n^\gamma}{z}\frac{\mathrm{d}[\delta\eta^L]_l{}^m}{\mathrm{d}z} = -\,[\delta\gamma^N_{L\Phi}]_l{}^m + \frac{[\underline{\eta}^N]_\beta{}^\alpha}{\eta^N_{\mathrm{eq}}}[\delta\gamma^N_{L\Phi}]_l{}^m{}_\alpha{}^\beta + \frac{[\delta\eta^N]_\beta{}^\alpha}{2\eta^N_{\mathrm{eq}}}[\gamma^N_{L\Phi}]_l{}^m{}_\alpha{}^\beta - \frac{1}{3}\Big\{\delta\eta^L,\, \gamma^{L\Phi}_{L^{\tilde{c}}\Phi^{\tilde{c}}} + \gamma^{L\Phi}_{L\Phi}\Big\}_l{}^m - \frac{2}{3}[\delta\eta^L]_k{}^n\Big([\gamma^{L\Phi}_{L^{\tilde{c}}\Phi^{\tilde{c}}} - [\gamma^{L\Phi}_{L\Phi}]\Big)_n{}^k{}_l{}^m - \frac{2}{3}\Big\{\delta\eta^L,\, \gamma_{\mathrm{dec}}\Big\}_l{}^m + [\delta\gamma^{\mathrm{back}}_{\mathrm{dec}}]_l{}^m$$

Numerical Examples

- Minimal model of Resonant τ-Genesis
  [A. Pilaftsis, PRL95 (2005) 081602]
- $O(3)$ symmetric heavy-neutrino sector at $\mu_X \sim M_{\rm GUT}$ broken to almost exact $U(1)_{L_e+L_\mu} \times U(1)_{L_\tau}$ at m_N.
- Mass splitting Δm_N from RG evolution to the scale m_N

$$\boldsymbol{M}_N \;=\; m_N\mathbf{1} \;-\; \frac{m_N}{8\pi^2}\ln\left(\frac{\mu_X}{m_N}\right)\mathrm{Re}\big[\boldsymbol{h}^\dagger(\mu_x)\boldsymbol{h}(\mu_X)\big]$$

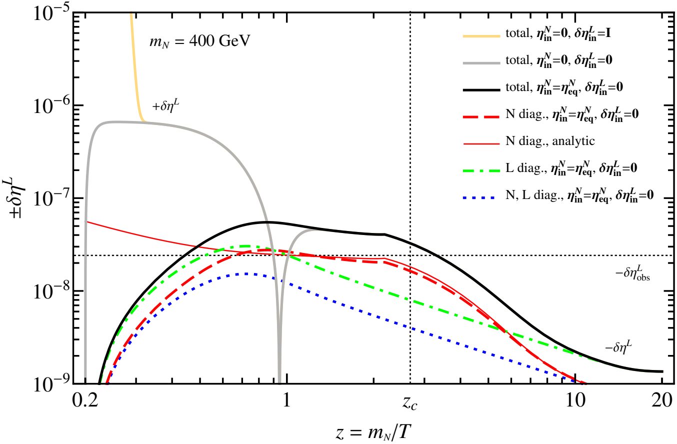
$$\boldsymbol{h} \;=\; \begin{pmatrix} 0 & ae^{-i\frac{\pi}{4}} & ae^{i\frac{\pi}{4}} \\ 0 & be^{-i\frac{\pi}{4}} & be^{i\frac{\pi}{4}} \\ 0 & 0 & 0 \end{pmatrix} + \boldsymbol{\delta h} \qquad \boldsymbol{\delta h} \;=\; \begin{pmatrix} \epsilon_e & 0 & 0 \\ \epsilon_\mu & 0 & 0 \\ \epsilon_\tau & \kappa_1 e^{-i(\frac{\pi}{4}-\gamma_1)} & \kappa_2 e^{-i(\frac{\pi}{4}-\gamma_2)} \end{pmatrix}$$

- Agreement with light neutrino masses $\boldsymbol{M}_\nu \;=\; -\frac{v^2}{2}\boldsymbol{h}\boldsymbol{M}_N^{-1}\boldsymbol{h}^{\rm T}$ for $m_N \sim 10^2$ GeV.

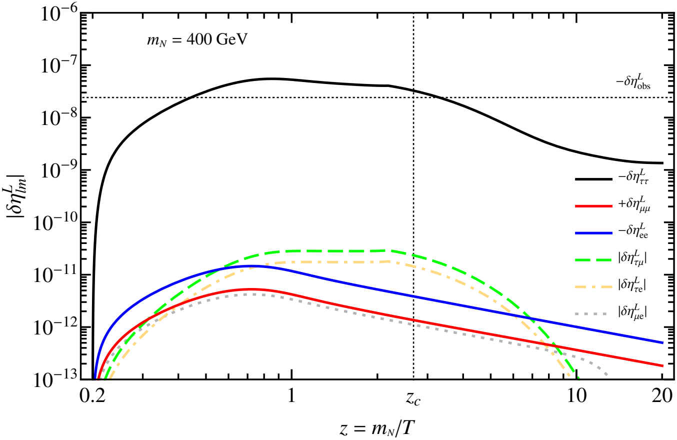
m_N	400 GeV
γ_1	$\pi/3$
γ_2	0
κ_1	2.4×10^{-5}
κ_2	6×10^{-5}
a	$(4.93 - 2.32\,i) \times 10^{-3}$
b	$(8.04 - 3.79\,i) \times 10^{-3}$
ϵ_e	5.73×10^{-8}
ϵ_μ	4.3×10^{-7}
ϵ_τ	6.39×10^{-7}

Observable	Model	Exp. Limit
$\mathrm{BR}(\mu \to e\gamma)$	1.9×10^{-13}	$< 5.7 \times 10^{-13}$
$\mathrm{BR}(\tau \to \mu\gamma)$	1.6×10^{-18}	$< 4.4 \times 10^{-8}$
$\mathrm{BR}(\tau \to e\gamma)$	5.9×10^{-19}	$< 3.3 \times 10^{-8}$
$\mathrm{BR}(\mu \to 3e)$	9.3×10^{-15}	$< 1.0 \times 10^{-12}$
$\mathrm{R}^{\mathrm{Ti}}_{\mu\to e}$	2.9×10^{-13}	$< 6.1 \times 10^{-13}$
$\mathrm{R}^{\mathrm{Au}}_{\mu\to e}$	3.2×10^{-13}	$< 7.0 \times 10^{-13}$
$\mathrm{R}^{\mathrm{Pb}}_{\mu\to e}$	2.2×10^{-13}	$< 4.6 \times 10^{-11}$
$\lvert\Omega\rvert_{e\mu}$	1.8×10^{-5}	$< 7.0 \times 10^{-5}$
$\langle m \rangle$ [eV]	3.8×10^{-3}	$< (0.11\text{–}0.25)$

Numerical Results



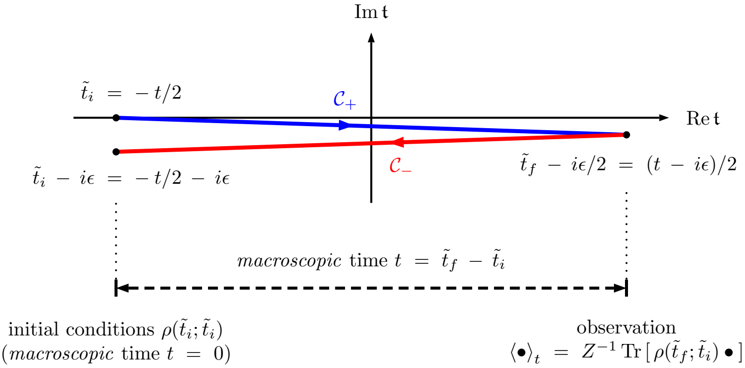
Numerical Results



Field-Theoretic Transport Phenomena

Schwinger-Keldysh Closed-Time Path (CTP) Formalism: a means to calculate (statistical) expectation values $\mathrm{Tr}\,\rho\,\bullet$ in QFT.

[J. S. Schwinger, J. Math. Phys. **2** (1961) 407;
L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47** (1964) 1515 [Sov. Phys. JETP **20** (1965) 1018]]



[See P. Millington, A. Pilaftsis, PRD88 (2013) 085009 and P. S. B. Dev et al., arXiv:1410.6434 and extensive list of references therein, inc. alternative approaches (many authors).]

Field-Theoretic Transport Phenomena

Kadanoff-Baym Equations

Partially inverting the CTP Schwinger-Dyson equation gives the **Kadanoff-Baym equations**
[L. Kadanoff, G. Baym, *Quantum Statistical Mechanics*, Benjamin, New York (1962)]

$$\left(p^2 - |\boldsymbol{m}|^2 \cdot + \mathbf{\Pi}_{\mathcal{P}} \star \right)\mathbf{\Delta}_{\gtrless} = -\frac{1}{2}\left(\mathbf{\Pi}_{>} \star \mathbf{\Delta}_{<} - \mathbf{\Pi}_{<} \star \mathbf{\Delta}_{>} + 2\,\mathbf{\Pi}_{\gtrless} \star \mathbf{\Delta}_{\mathcal{P}}\right)$$

Use [Millington and Pilaftsis (2013)] to obtain the **rate equation**:

$$\frac{\mathrm{d}\boldsymbol{n}(t,\mathbf{X})}{\mathrm{d}t} - \int_{p,\,p'}^{(X)} (\mathbf{p}^2 - \mathbf{p}'^2)\,\mathbf{\Delta}_{<} - \int_{p,\,p'}^{(X)} \Big([|\boldsymbol{m}|^2,\ \mathbf{\Delta}_{<}] - [\mathbf{\Pi}_{\mathcal{P}},\ \mathbf{\Delta}_{<}]_{\star}\Big)$$
$$= -\frac{1}{2}\int_{p,\,p'}^{(X)} \Big(\{\mathbf{\Pi}_{>},\ \mathbf{\Delta}_{<}\}_{\star} - \{\mathbf{\Pi}_{<},\ \mathbf{\Delta}_{>}\}_{\star} + 2\,[\mathbf{\Pi}_{<},\ \mathbf{\Delta}_{\mathcal{P}}]_{\star}\Big)\,.$$

Require an **approximation scheme** to make this **tractable**.

Field-Theoretic Transport Phenomena

Physical Observables

- Kadanoff-Baym approaches give equations for **propagators** and **self-energies**, **not number densities**.
- Often perform a **gradient expansion** in time-derivatives and use a **Kadanoff-Baym ansatz** for the dressed heavy-neutrino propagators

$$i\mathbf{\Delta}^{N}_{<}(k,t) \;=\; 2\pi\delta(k^2 - m_N^2)\boldsymbol{n}^N(\mathbf{k},t)$$

  as perturbative loopwise truncations are usually not possible.
- The latter **is** possible [Millington and Pilaftsis (2013)] and we may truncate two ways
  1. spectrally: truncating the external propagator — decides what we count
  2. statistically: truncating the self-energy — decides what processes drive the evolution

Field-Theoretic Transport Phenomena

Perturbative Loopwise Truncation

Spectrally truncate the heavy-neutrino equation

$$\frac{\mathrm{d}[n^N]_\alpha{}^\beta}{\mathrm{d}t} = \int_k \theta(k_0)\Big\{ -\, i\,\big[M_N^2,\, i\Delta^{N,\,0}_{<}(k,t)\big]_\alpha{}^\beta - \frac{1}{2}\Big(\big\{i\Pi^N_<(k),\, i\Delta^{N,\,0}_>(k,t)\big\}_\alpha{}^\beta - \big\{i\Pi^N_>(k),\, i\Delta^{N,\,0}_<(k,t)\big\}_\alpha{}^\beta\Big)\Big\}$$

In a Markovian and homogeneous approximation, the spectrally free heavy-neutrino propagator is unambiguous:

$$[i\Delta^{N,\,0}_{\gtrless}(k,t)]_\alpha{}^\beta \;=\; 2\pi\delta(k^2 - m_N^2)\Big(\theta(\pm k_0)\delta_\alpha{}^\beta \;+\; [n^N(\mathbf{k},t)]_\alpha{}^\beta\Big)\;.$$

Appropriate to approximate the charged-lepton and heavy-neutrino propagators by their equilibrium quasi-particle forms:

$$i\Delta^{\Phi,\,\mathrm{eq}}_{\gtrless}(q) \;=\; 2\pi\delta(q^2 - M_\Phi^2)\,\big[\theta(\pm q_0) \;+\; n^\Phi_{\mathrm{eq}}(\mathbf{q})\big]\;,$$

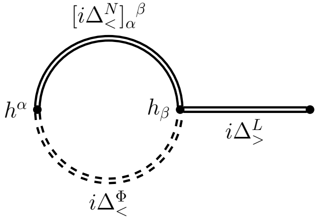
$$i\Delta^{L,\,\mathrm{eq}}_{\gtrless}(p) \;=\; 2\pi\delta(p^2 - M_L^2)\,\big[\theta(\pm p_0) \;+\; \theta(p_0)n^L_{\mathrm{eq}}(\mathbf{p}) \;+\; \theta(-p_0)\overline{n}^L_{\mathrm{eq}}(\mathbf{p})\big]\;.$$

Field-Theoretic Transport Phenomena

Perturbative Loopwise Truncation

Statistically truncate the equation for the asymmetry

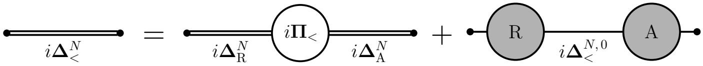
$$\frac{\mathrm{d}\delta n^L}{\mathrm{d}t} \supset -\, i \int_{k,k',\,p,\,q} \theta(p_0 + k_0' - q_0)(2\pi)^4 \delta^{(4)}(p - k + q) \times \Big[h_\beta h^\alpha \Big([\Delta^N_<(k,k',t)]_\alpha{}^\beta \, \Delta^{\Phi,\,\mathrm{eq}}_>(q)\, \Delta^{L,\,\mathrm{eq}}_>(k'-q) - [\Delta^N_>(k,k',t)]_\alpha{}^\beta \, \Delta^{\Phi,\,\mathrm{eq}}_>(q)\, \Delta^{L,\,\mathrm{eq}}_>(k'-q) \Big) - \widetilde{C}.c. \Big]$$



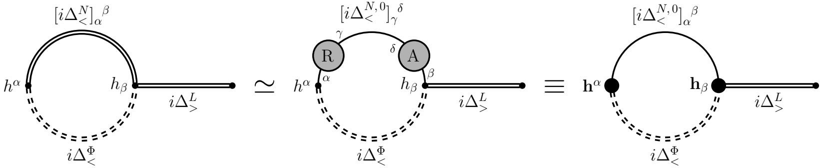
Field-Theoretic Transport Phenomena

Mixing Phenomena

Resummed heavy-neutrino propagator



Inserting in the equation for the asymmetry

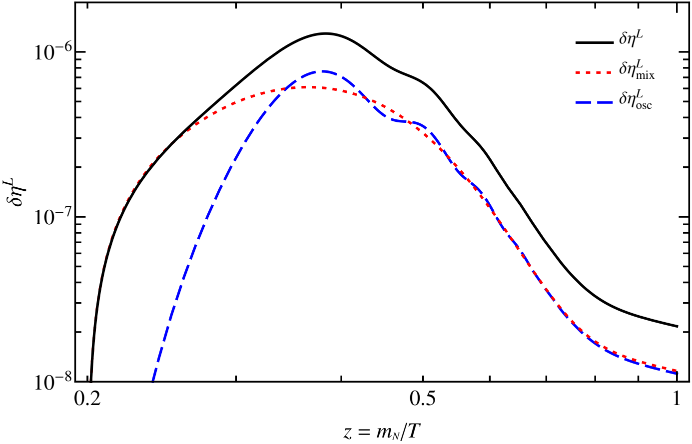


Such **absorptive transitions** are implicitly **discarded** by **quasi-particle ansatz** for the **dressed** heavy-neutrino propagators.

Exact equivalence with inclusion of effective Yukawa couplings in the semi-classical treatment.

Field-Theoretic Transport Phenomena

Mixing and Oscillations



Combination $\delta\eta^L = \delta\eta^L_{\text{osc}} + \delta\eta^L_{\text{mix}}$ yields a factor of 2 enhancement compared to the isolated contributions for weakly-resonant RL.

Conclusions

- **Resonant Leptogenesis** (and in particular **Resonant ℓ-Genesis**) scenarios provide predictive models that are **testable** at current and future experiments at the energy and intensity frontiers.
- **Three** physically-distinct **flavour effects**:
  1. **Oscillations** between heavy-neutrino flavours ($\delta\eta^N$)
  2. **Mixing** between heavy-neutrino flavours ($\underline{\eta}^N$ and $\delta\gamma^N_{L\Phi}$)
  3. **(De)coherence** effects in the charged-lepton sector
- Final asymmetry may be **enhanced** by as much as **an order of magnitude** compared to partially flavoured treatments, expanding viable parameter space.
- Presented a **fully-flavour covariant** treatment of semi-classical transport phenomena.
- Mixing **and** oscillations **both** present in "first-principles" approaches when embedding the **flavour-covariant formalism** into a **perturbative non-equilibrium quantum field theory**.