# Playing with quantum gravity in the laboratory

Jacob D. Bekenstein
Hebrew University of Jerusalem
and
University College London

Seminar - University of Liverpool - 14 May, 2014





Theoretical discussion about the quantum theory of gravity is a least 70 years old. Meanwhile technology in the service of physics has developed by leaps and bounds. The discovery of the Higgs boson and of the E-polarization of the cosmological microwave background are just two examples of technical achievements in this direction. But so far none of quantum gravity's touted features have been put in evidence in the lab. Plans for experiments? Plenty, as you will see. Hard results? Not really. So even though many would class quantum gravity as the number-one open problem in theoretical physics, there is no general agreement as to what the true theory of quantum gravity looks like.

# Planck length - Planck scale

M. Planck, *Sitzungsberichte der Königlich Preußischen Akademie der Wisseschaften zu Berlin* - Erster Halb band (Berlin 1899).

$$\ell_P = (\hbar G/c^3)^{1/2} = 1.616 \times 10^{-33}\,\mathrm{cm}$$

$$t_P = (\hbar G/c^5)^{1/2} = 5.391 \times 10^{-44}\,\mathrm{s}$$

$$m_P = (\hbar c/G)^{1/2} = 2.177 \times 10^{-5}\,\mathrm{g}$$

Max Planck
(1858-1947)



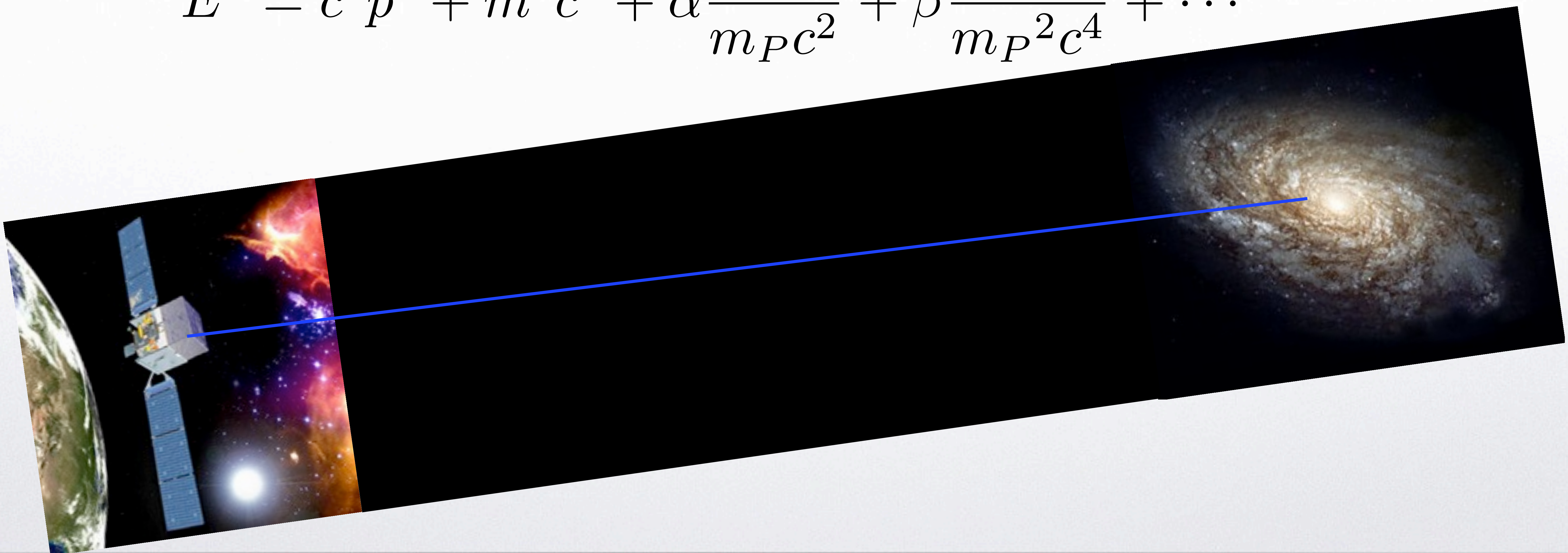


In 1899 Max Planck, fresh from his identification of the quantum of action in black body radiation theory, noticed that from his h and G and c you can build fundamental units of length, time and mass – fundamental in that they do not depend on any specific particle.   At some point it came to be speculated that \ell_P is the scale at which quantum effects become noticeable in gravitational physics.  But it is such a tiny length that the issue was all but forgotten.    Interest in the issue revived, together with general interest in gravity physics, starting in the 1950's.  But let's jump to our decade.  I would first like to mention, in passing, three types of experiments which have been proposed, in the last decade, to test features of quantum gravity.

# Modified dispersion relation and gamma ray bursts

$$E^2 = c^2p^2 + m^2c^4$$

$$E^2 = c^2p^2 + m^2c^4 + \alpha\frac{E^3}{m_Pc^2} + \beta\frac{E^4}{{m_P}^2c^4} + \cdots$$





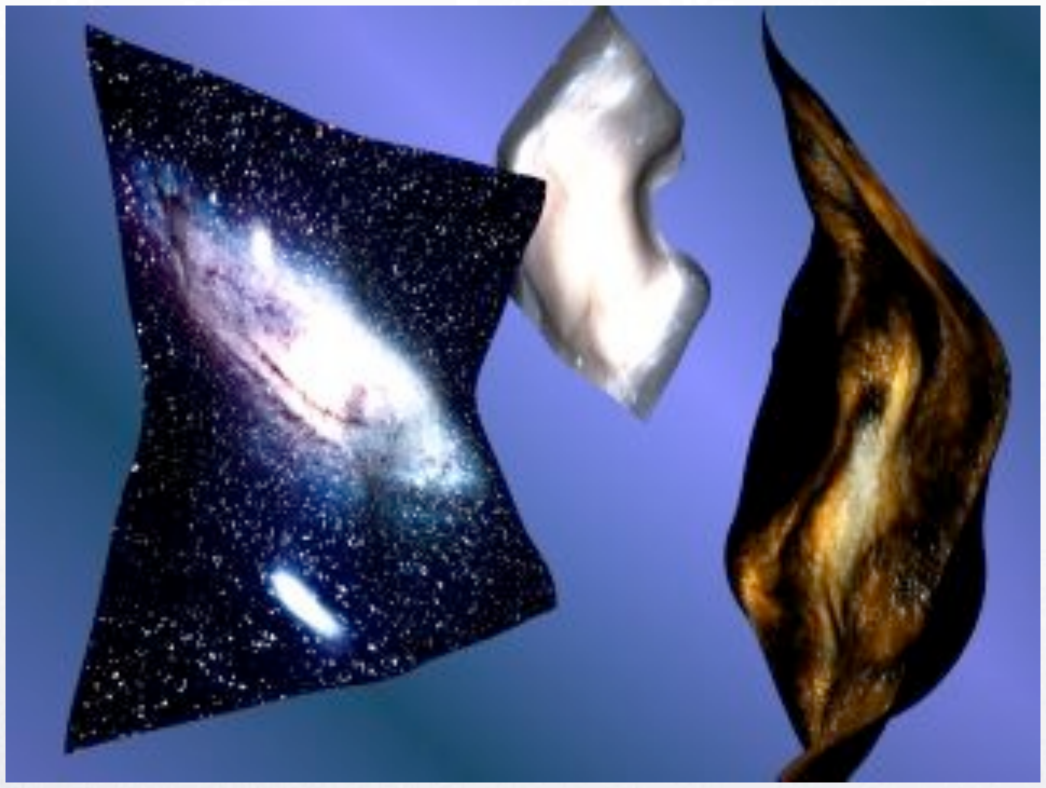


People have speculated that quantum gravity may distort the standard energy-momentum dispersion relation into, say, …. with $\alpha, \beta, \cdots$ a set of dimensionless constants. Evidently this relation clashes with Lorentz invariance, a principle likely to fall victim to quantum gravity. One consequence of the proposed distortion is that the speed of particles does not asymptote to $c$ for $E\gg mc^2$, but rather remains energy dependent. This prediction can be tested by looking at the duration, at Earth, of a gamma ray burst as a function of energy. Gamma ray bursts, comprising photons with a gamut of energies, come from very distant galaxies, and the corresponding long timelines allow the energy dependent speed to spread the bursts in time. Since there must be an initial spread, the said measurements can only determine an upper bound on the temporal spread. Data obtained by the Fermi $\gamma$-ray satellite show that $\alpha$, if it does does not vanish identically is constrained be no bigger than order unity. But if $\alpha$ vanishes identically, nothing useful can be said about $\beta$. Evidently the measurements do not settle anything---yet.

# Quantum black holes in the LHC?

$$m_P\, c^2 = (\hbar c^5/G)^{1/2} \approx 10^{19}\, \mathrm{GeV}$$

D dimensional spacetime (D>4)



$$(\ell_P^{(D)})^{D-2} = \ell_P{}^2\, G^{(D)}/G$$

$$\ell_P^{(D)} \gg (\hbar G/c^3)^{1/2}$$

$$m_P^{(D)}\, c^2 \ll 10^{19}\, \mathrm{GeV}$$



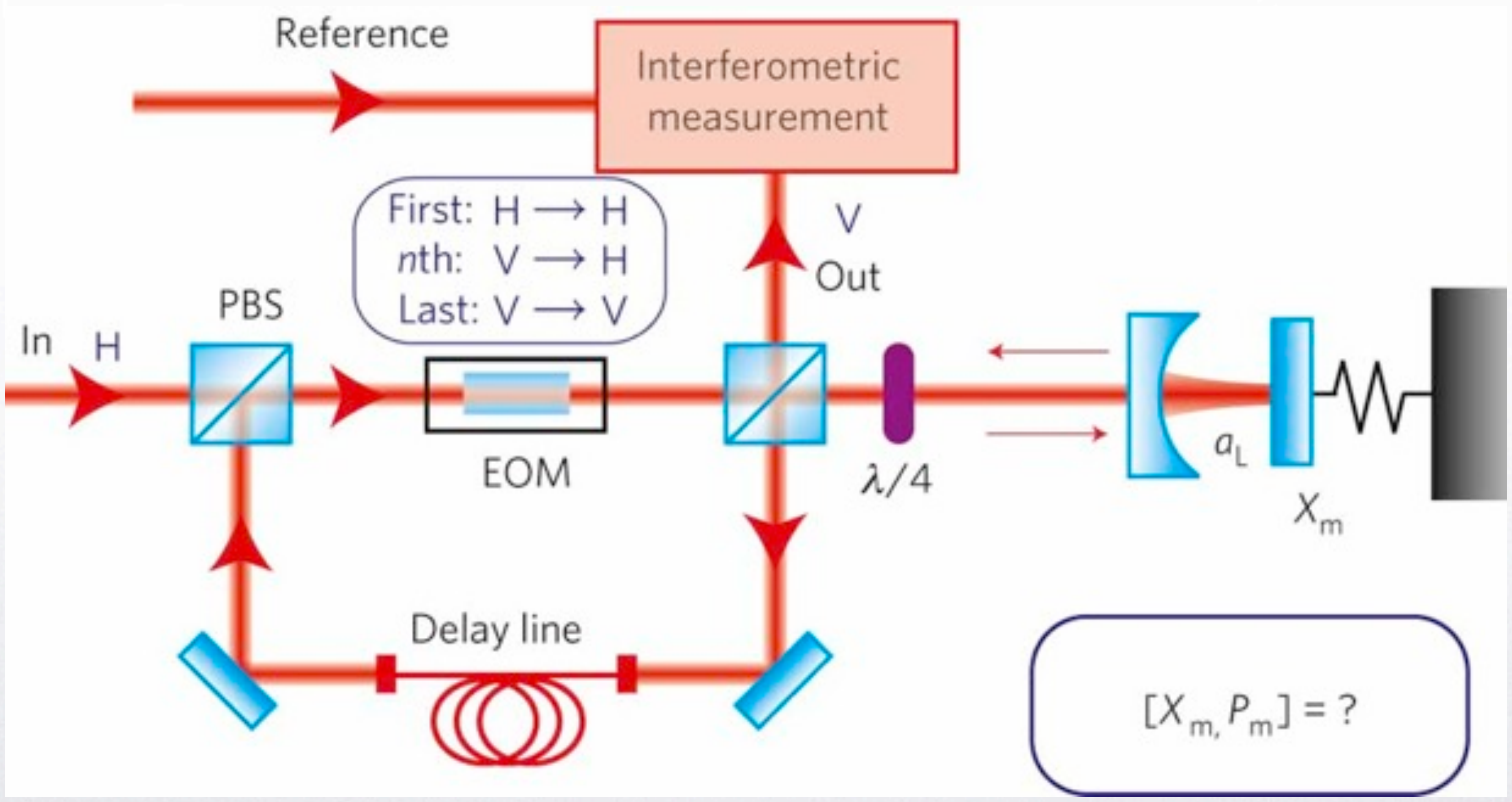


A second approach is to search for quantum black holes in the debris of high energy collisions, e.g. in the Large Hadron Collider (LHC) in Geneva. A quantum black hole is one having a mass near Planck mass \m_P. In reality neither LHC nor any presently imagined accelerator can access the corresponding energy, about $10^{19} $GeV. What the investigators have in mind is the string theory-inspired scenario whereby the fermions and gauge fields which make up the matter we perceive, and its interactions, are confined to a four-dimensional brane, a subspace, in a world with $D>4$ dimensions. Gravity pervades the $D $-dimensional space-time. In such a world the true Planck length $\ell_P^{(D)}$, the critical scale at which quantum effects become strong for gravity, is related to the nominal Planck length $(\hbar G/c^3)^{1/2}$ ($G$ is the measured Newton constant) by (\ell_P^{(D)})^{D-2}=\ell_P{}^2 G^{(D)}/G where $G^{(D)}$ is the gravitational constant in $D$ dimensions. It is obviously possible that $\ell_P^{(D)}\gg \ell_P$. Since $m_P^{(D)}=\hbar /(c\,\ell_P^{(D)})$, the physical Planck mass in the brane scenario can be much below $(\hbar G/c)^{1/2}$ and the LHC may be able to access the corresponding energy and produce black holes. Thus far no evidence of black hole formation has surfaced at the LHC. This might mean that $G^{(D)}$ is not very large compared to $G (\ell_P^{(D)})^{D-4}$, or that space-time is four dimensional after all. From now on I am going to assume that spacetime is 4-D.

# Modified uncertainty principle: a laboratory experiment

$$\Delta x \, \Delta p \geq \hbar/2 \qquad\qquad [\hat{x}, \hat{p}] = \imath\hbar$$

**I. Pikovski**, **M. R. Vanner**, **M. Aspelmeyer**, **M. S. Kim** & **Č. Brukner**, *Nature Physics* **8**, 393–397 (2012)








The Heisenberg uncertainty relation is closely bound up with the form of the canonical commutator. It has been speculated that due to quantum gravity there might be a correction to the canonical commutator, for example ... That would give a modified uncertainty relation. Today the bounds on beta_0 are of order 10^(33). A first order correction would have its coefficient bounded by 10^(10). The following experiment suggested by Pikovski et al. A Planck scale mass constitutes an harmonic oscillator attached to the wall of an optical cavity. A probe beam enters several times at particular phases during the oscillator's cycle. The commutator characteristics are thus mapped onto the phases of the beam. This beam is then measured interferometrically. This experiment has not yet run. It is expected to allow reduction of the bound on beta_0 to order unity.

# Some quantum gravity theories

- Canonical quantum gravity
- Loop quantum gravity
- String theory - branes - extra large dimensions
- Faddeev quantum gravity
- Causal dynamical triangulations
- Horava-Lifshitz gravity
- others





The dearth of experimental results about quantum gravity explains why lots of quantum gravity schemes survive and coexist. Here is a partial list of frameworks for quantum gravity, old and new. Loop quantum gravity is a background independent canonical theory which can also be construed as a quantization of Einstein theory using variables that are more like electric and magnetic fields of gauge theory than like metric. In Faddeev gravity theory, the Lagrangian is quadratic in first order derivatives of 10 4-D vectors. This theory is classically equivalent to GR, but would lead to a different quantum gravity from the above. Causal dynamical triangulations is another background independent theory which starts by dividing up spacetime into simplices which are then stacked together in accordance with a causality rule. Calculations in this theory are mostly numerical. Horava gravity gives up local Lorentz invariance for renormalizability. Many ingenious ideas, but none of them is experimentally tested. It would be be neat for the subject to have some experimental test.

# Quantum foam (Wheeler 1955)

John A. Wheeler
(1911–2008)

- On sufficiently small scale, space is not smooth and has a complex topology (quantum foam)
- Rapid temporal fluctuations of the 3-geometry
- The critical scale - Planck's $\ell_P = (\hbar G/c^3)^{1/2} = 1.616 \times 10^{-33}\,\mathrm{cm}$





At least some of those theories I mentioned should predict the following set of features of quantum gravity (conjectured by Wheeler):

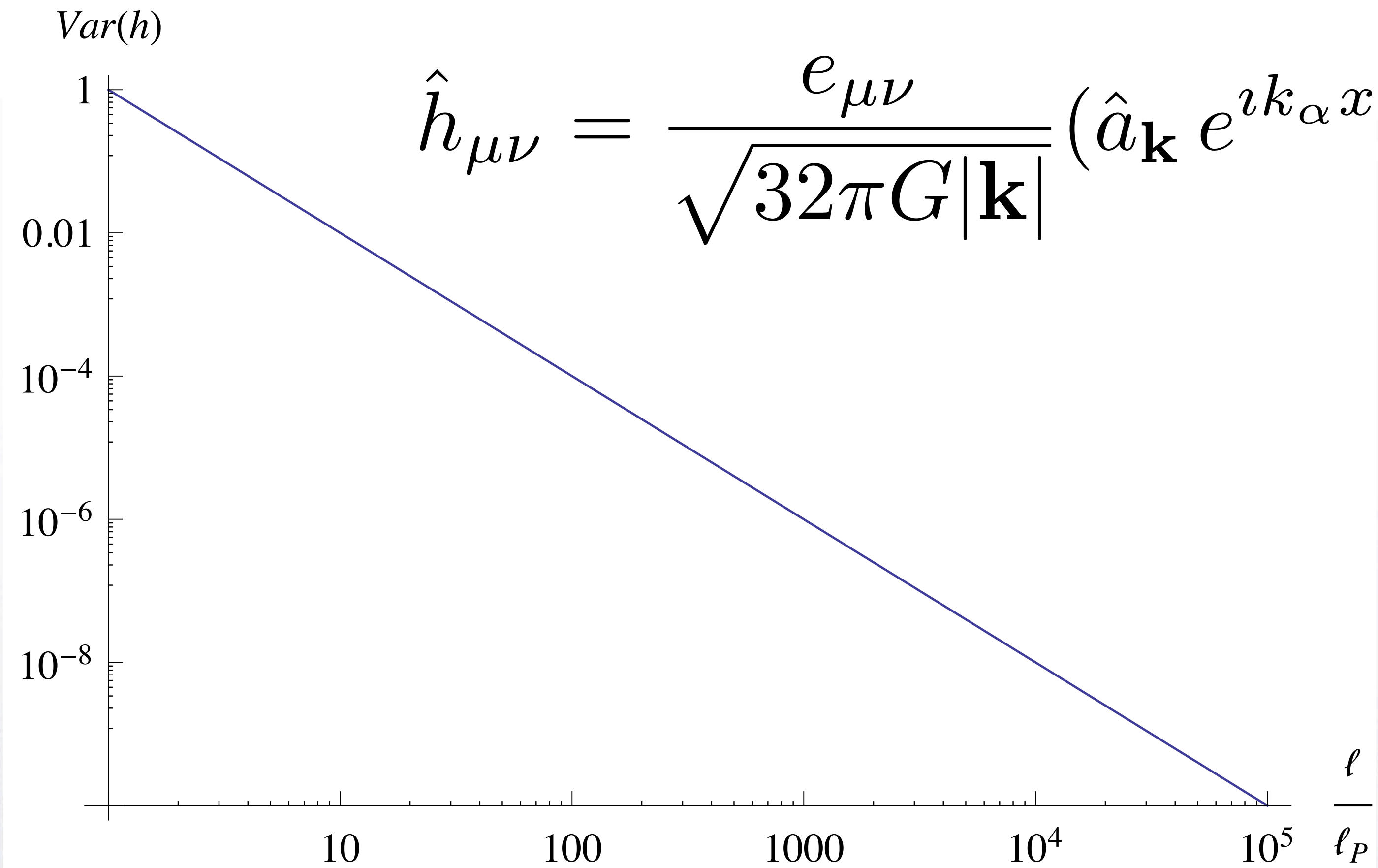
a) space below some critical length scale becomes not-smooth, and may exhibit a multiconnected topology.
b) for a given observer, the space geometry fluctuates rapidly on that same scale
c) The critical scale at which non-smoothness emerges is Planck's.

Why is this?

# quantum linear gravity

$$\hat{g}_{\mu\nu} = \eta_{\mu\nu} + \hat{h}_{\mu\nu}$$

$$\hat{h}_{\mu\nu} = \frac{e_{\mu\nu}}{\sqrt{32\pi G|\mathbf{k}|}}(\hat{a}_{\mathbf{k}}\, e^{\imath k_\alpha x^\alpha}$$








While there is no agreement on how to generically quantize gravity, few would deny that we know how to quantize weak gravitational fields. One separates the metric of spacetime into a Minkowski background plus a perturbation h. Einstein equations of gravitation, when linearized in h (with choice of gauge) tell us that h obeys the wave equation familiar from E&M. One quantizes h as one would the vector potential in E&M; there is need to pick a gauge that simplifies the equation and the energy momentum tensor. One thing to do is to estimate the fluctuations of h in the vacuum (no gravitons) as a function of spatial resolution. This is seen in the graph. Notice that the variance of h approaches unity as the resolution approaches the Planck scale. In pictures ...

# Achilles' heel - localization

**elementary particle, mass** $m$ $\quad \Delta x \sim \ell_P \quad \Delta p \sim \frac{\hbar}{\ell_P}$

$$K = \sqrt{m^2c^4 + c^2(\hbar/\ell_P)^2} - mc^2 \sim c\hbar/\ell_P = 10^{19}\,\mathrm{GeV}$$

**macroscopic mass** $M$ $\quad \Delta p \sim \frac{\hbar}{\ell_P} \quad \Delta v \sim \frac{\hbar}{M\ell_P}$

$$\frac{\tau\hbar}{M\ell_P} < \ell_P \qquad \tau < \frac{M}{m_P c}\ell_P \approx 10^{-32}\,\mathrm{s}$$

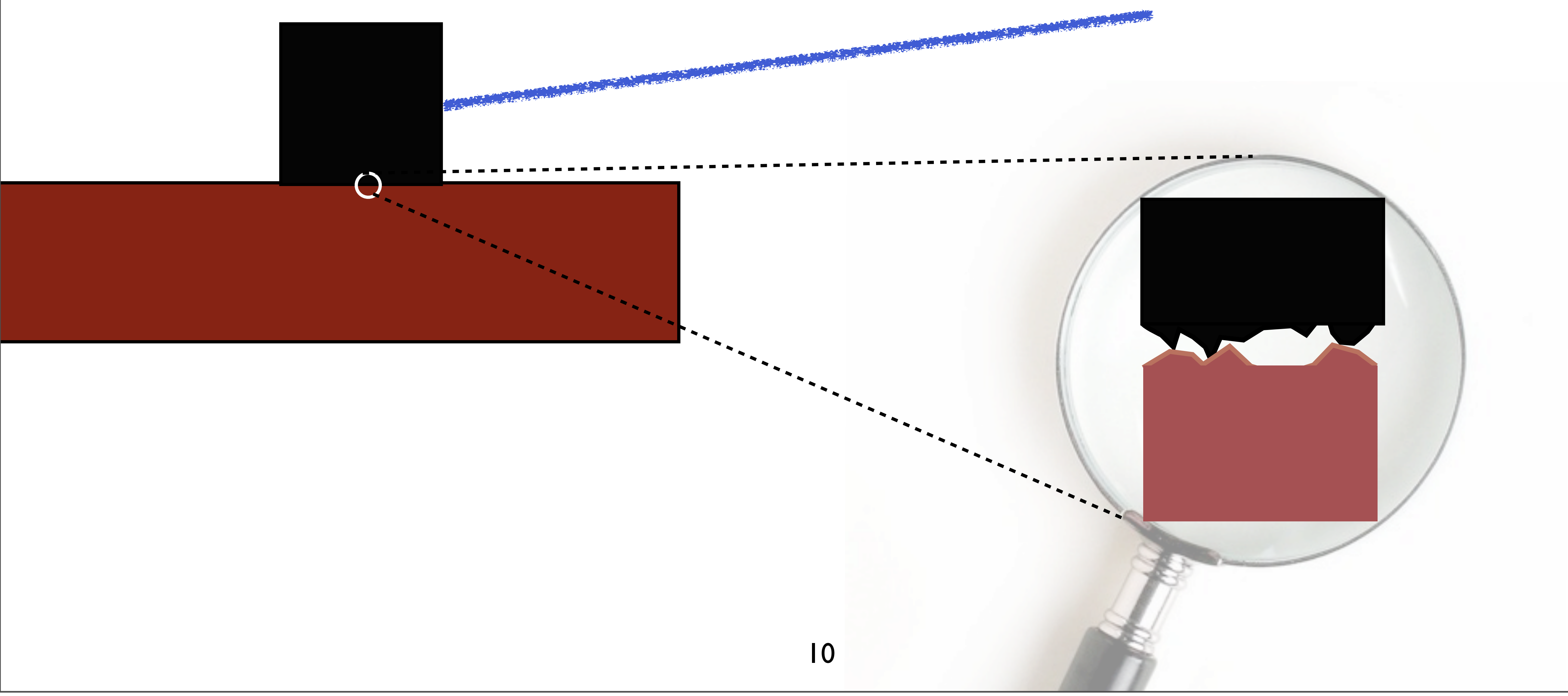




Since quantum foam should arise quite generically, we would like to expose it experimentally. Here is the problem. Suppose one tries to probe spacetime on Planck scale with an elementary particle. Such microscopic probes require energies 8 orders above those of the most energetic cosmic rays, which themselves are very far from being imitated by accelerators. Things are easier at larger m. Wouldn't we be doing better with macroscopic probes? Even before asking how this would be done consider the following. A macroscopic probe needs to be activated only over extremely short times (assuming M=1 ton), beyond all foreseeable technological temporal resolution. In light of these examples it is fair to say that the Achilles' heel of many procedures for bringing out quantum gravity's experimental consequences is in the requisite amount of localization of the probes.
The conclusion can only be one: to probe Planck scale roughness of spacetime, one has to avoid localization of the probes, either before or after any measurement.

# What do we want to achieve ?

Translate a macroscopic probe's c.m. by a controllable distance of order Planck's length, starting from an unspecified point - all this in the initial frame of rest of the c.m.



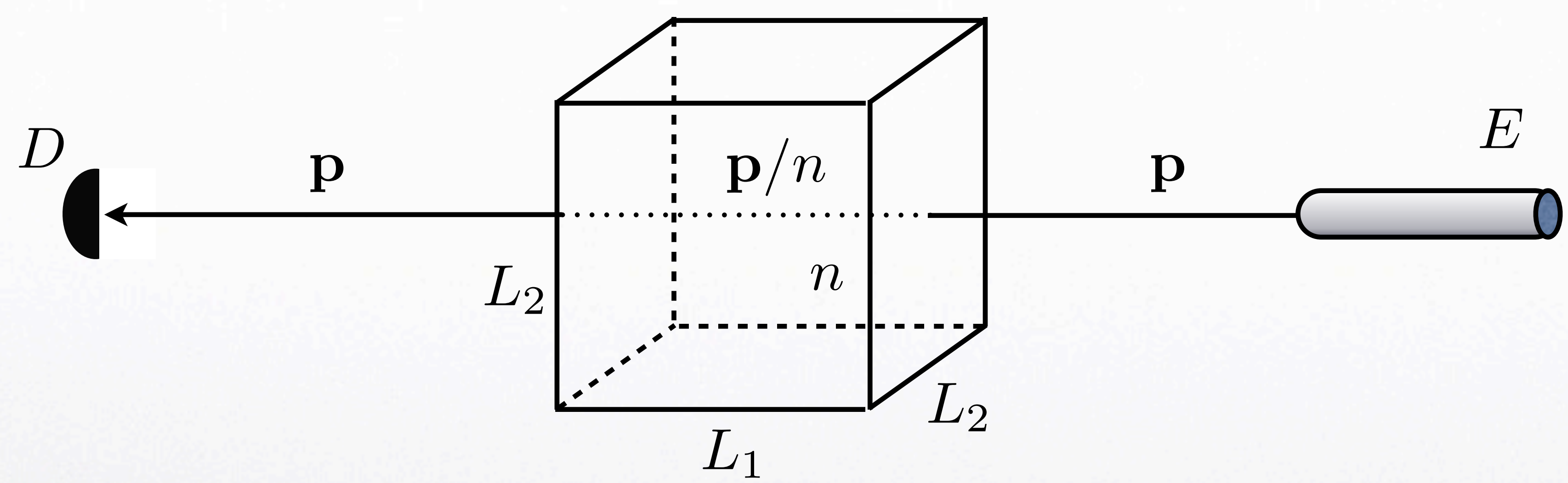
10



So what we want to achieve is ...... This gets us out of having to localize the probe as part of the experiment. But how does this help us? The idea is that shifting a macroscopic object by distances of order Planck's length should be qualitatively different from shifting by a macroscopic distance (over which spacetime looks smooth). In some theories --- notably loop quantum gravity --- there is no such thing as smaller than Planck distances. More generally can illustrate with the following analogy. Think of dragging a block of wood across the floor. If over a macroscopic time we move it a macroscopic distance – 1 cm say – we are up against dynamic friction. If we drag it in the same time interval over an atomic distance the motion is opposed by static friction – much larger. There is direct contact between asperities of atomic scale on the boundaries; by contrast, when the shift is big, those irregularities are literally melted away. By analogy we expect a shift by a Planck length to be, in some sense, harder to perform than a much longer shift.

# An experiment to expose the quantum foam

JB, Phys. Rev. **D** 86, 124040 (2012); Found. Mod. Phys. ArXiv1301.4322.








Let me now describe an idea for an experiment based on these remarks.
First look at an idealized experiment.
An accurately rectangular (need not be cubic in shape) block of very transparent dielectric--either amorphous or crystalline of the cubic class (optically isotropic). The index of refraction is n. A single photon emitter sends an optical photon accurately normally to one face. A detector records the photon after traversal. Inside the block the photon has momentum reduced by a factor n. This is important, so let me show why.

# A photon's momentum

$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\mathbf{E} = \mathbf{E}_0 \exp(\imath \mathbf{k} \cdot \mathbf{r} - \imath \omega t)$$

$$c\,\mathbf{k} \times \mathbf{E} = \omega \mathbf{B}$$

$$c\,k = n\omega$$

$$n = \sqrt{\epsilon\mu}$$

$$\mathbf{H} = \mathbf{B}/\mu$$

$$\sqrt{\frac{\epsilon}{\mu}}\frac{\mathbf{k}}{k} \times \mathbf{E} = \mathbf{H}$$

$$\rho_p = \frac{|\mathbf{E} \times \mathbf{H}|}{4\pi c} = \sqrt{\frac{\epsilon}{\mu}}\frac{E^2}{4\pi c}$$

$$\frac{\rho_e}{\rho_p} = c\sqrt{\epsilon\mu} = c\,n$$

$$\mathbf{p}_{\text{in}} = \frac{\mathbf{p}_{\text{out}}}{n}$$

$$\rho_e = \frac{\epsilon E^2}{8\pi} + \frac{\mu H^2}{8\pi} = \frac{\epsilon}{4\pi}E^2$$

# Abraham-Minkowski controversy (1909 - 2010)

Max Abraham
(1875-1922)

$$\rho_p = \frac{|\mathbf{E}\times\mathbf{H}|}{4\pi c}$$

$$\mathbf{p} = \frac{\hbar\mathbf{k}_0}{n}$$

G. B. Walker, D. G. Lahoz, and G. Walker, Can. J. Phys. **53**, 2577 (1975).
W. She, J. Yu, and R. Feng, Phys. Rev. Lett. **101**, 243601 (2008).

Hermann Minkowski
(1864-1909)

$$\rho_p = \frac{|\mathbf{D}\times\mathbf{B}|}{4\pi c}$$

$$\mathbf{p} = n\hbar\mathbf{k}_0$$

R. V. Jones and J. C. S. Richards, Proc. R. Soc. A **221**, 480 (1954); A. Ashkin and J. M. Dziedzic, Phys. Rev. Lett. **30**, 139 (1973); R. V. Jones and B. Leslie, Proc. R. Soc. A **360**, 347 (1978); A. F. Gibson, M. F. Kimmitt, A. O. Koohian, D. E. Evans, and F. D. Levy, Proc. R. Soc. A **370**, 303 (1980); M. Kristensen and J. P. Woerdman, Phys. Rev. Lett. **72**, 2171 (1994).
G. K. Campbell, A. E. Leanhardt, J. Mun, M. Boyd, E. W. Streed, W. Ketterle, and D. E. Pritchard, Phys. Rev. Lett. **94**, 170403 (2005).

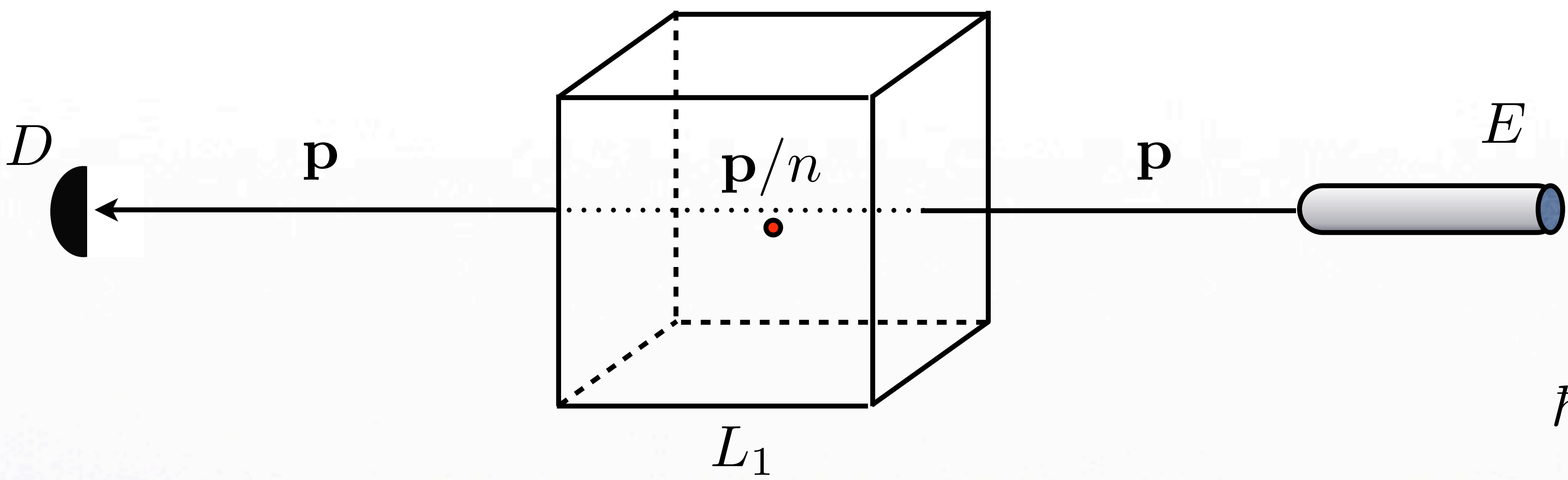
S. M Barnett, PRL 104, 070401 (2010)





Actually the result touches on a century old controversy – Minkowski vs Abraham – about the form of the electromagnetic energy-momentum tensor in matter. Experiments meant to decide between them were carried out, at least since the 70's. Some supported Abraham and some Minkowski. The mystery may have been solved by a 2010 paper of Barnett: both are right, Abraham's momentum density refers to the kinetic momentum and Minkowski's to the canonical momentum. I use Abraham's form. With Minkowski's form a slight change in the formulas, and the sign of the effect may be opposite. But the order of magnitude is the same.

# Translation of the c. m. in initial rest frame




$$L_1 = 1\,\mathrm{mm}$$

$$M = 0.15\,\mathrm{g}$$

$$\hbar\omega = 2.78\,\mathrm{eV} \qquad \lambda = 445\,\mathrm{nm}$$

$$\Delta X = 1.98 \times 10^{-33}\,\mathrm{cm}$$

$$\Delta p = \frac{\hbar\omega}{c} - \frac{\hbar\omega}{cn} = \frac{\hbar\omega}{c}\left(1 - \frac{1}{n}\right) = P$$

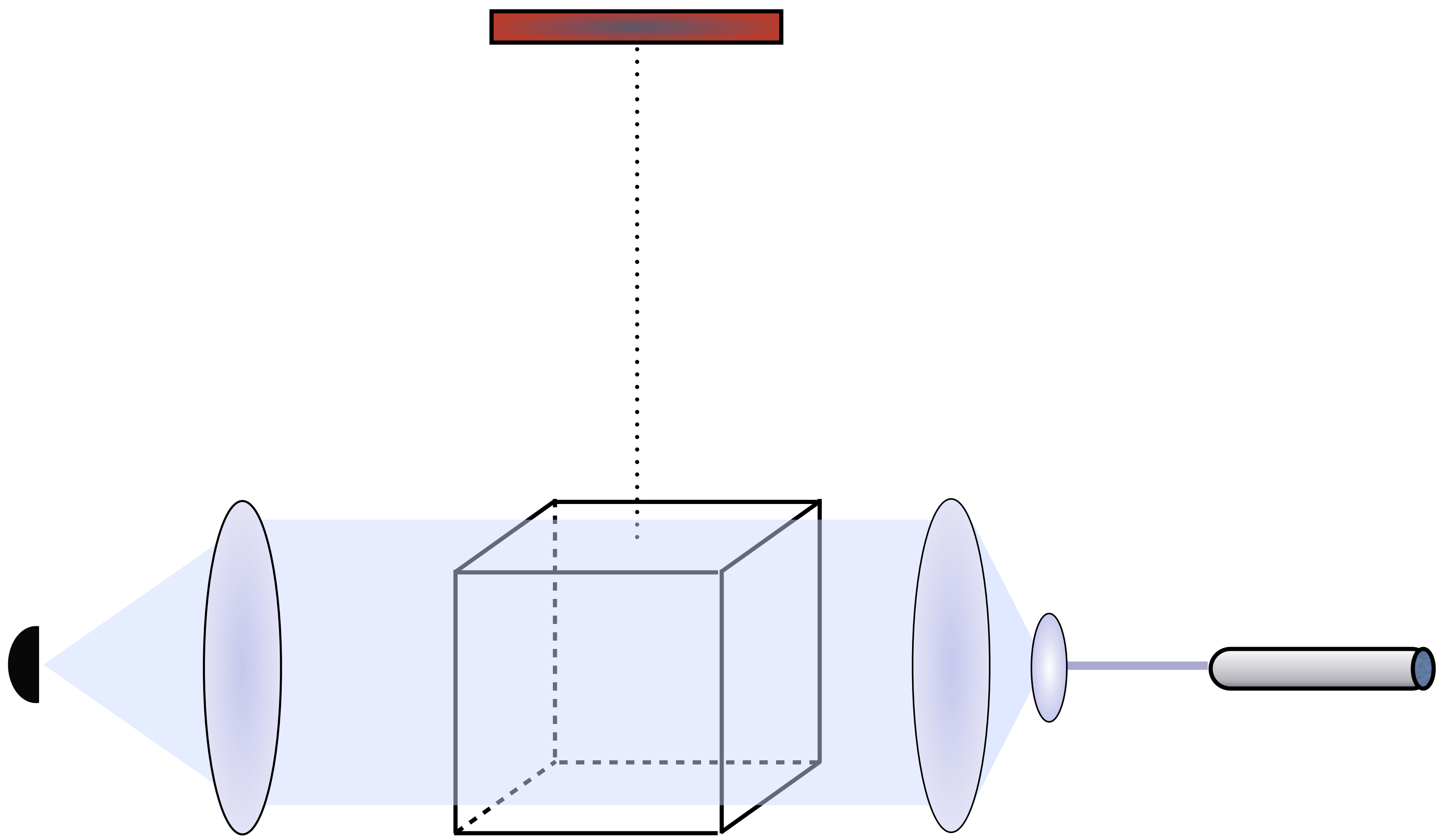
$$\Delta X = \frac{P}{M} \times \frac{nL_1}{c} = L_1 \frac{\hbar\omega}{Mc^2}(n-1)$$





The argument gives the translation of the c.m. of the block due to the passage of the photon. No knowledge of initial position of the block used – localization not needed. With suitable choice of parameters the translation can be made comparable with Planck's length. The question that we are going to face is that such a short translation may be impeded by spacetime fluctuations. What will then really happen to the photon? Before turning to those issues let me clarify some details about the experiment.

# Some necessary additions





Our idealized set up needs to face some realities. The block cannot be free in the lab, so the best alternative is to have it suspended by a very long light thread. Calculations show the restoring horizontal force to be negligible. If the incoming photon is transversally localized it will deposit momentum in a narrow tube within the block. This momentum will be transported out to the rest of the block by phonons, but they travel slowly, and so by the time the photon is out, only a small sector of the crystal partakes in the translation. To get a cleaner situation, one can use an optical system to broaden the beam so as to encompass the full breadth of the block, and one to refocus the outgoing beam onto the photon detector.

# Questions of compatibility

$\hat{\mathbf{r}}_i(t)$ and $\hat{\mathbf{p}}_i(t)$ subject to $[\hat{x}, \hat{p}_x] = \imath\hbar, \quad [\hat{x}, \hat{p}_y] = 0,$ etc.

$$\hat{\mathbf{R}}(t) \equiv \frac{1}{M}\sum_i m_i \hat{\mathbf{r}}_i(t) \qquad \hat{\mathbf{P}}(t) \equiv \sum_i \hat{\mathbf{p}}_i(t)$$

$$[\hat{\mathbf{R}}(t), \hat{\mathbf{P}}(t)] = \imath\hbar I$$

$$\Delta\hat{\mathbf{X}} \equiv \hat{\mathbf{R}}(t_f) - \hat{\mathbf{R}}(t_i)$$

What is $[\Delta\hat{\mathbf{X}}, \hat{\mathbf{P}}]$ ?

$$\hat{H} = \sum_i \frac{\hat{\mathbf{p}}_i(t)^2}{2m_i} + \sum_a \sum_{i \neq j} V_{(a)}\big(\hat{\mathbf{r}}_i(t) - \hat{\mathbf{r}}_j(t)\big) \implies \hat{\mathbf{P}} \text{ conserved}$$

$$\imath\hbar \frac{d\hat{\mathbf{R}}}{dt} = [\hat{\mathbf{R}}, \hat{H}] = \imath\hbar\, \hat{\mathbf{P}}/M \implies \hat{\mathbf{R}}(t) = \hat{\mathbf{R}}(t_i) + (t - t_i)\, \hat{\mathbf{P}}/M$$

$$[\Delta\hat{\mathbf{X}}, \hat{\mathbf{P}}] = 0$$





We have been talking about momentum of the c.m. and about its translation. Are these two variables not incompatible in quantum theory?

The argument shows that translation is compatible with momentum.

Another thing to note: the c. m. coordinate is canonically conjugate to the total block momentum, a conserved quantity. This serves as counter to any claim that it should not matter how much one translates the c.m. – that c. m. is devoid of importance in our context.

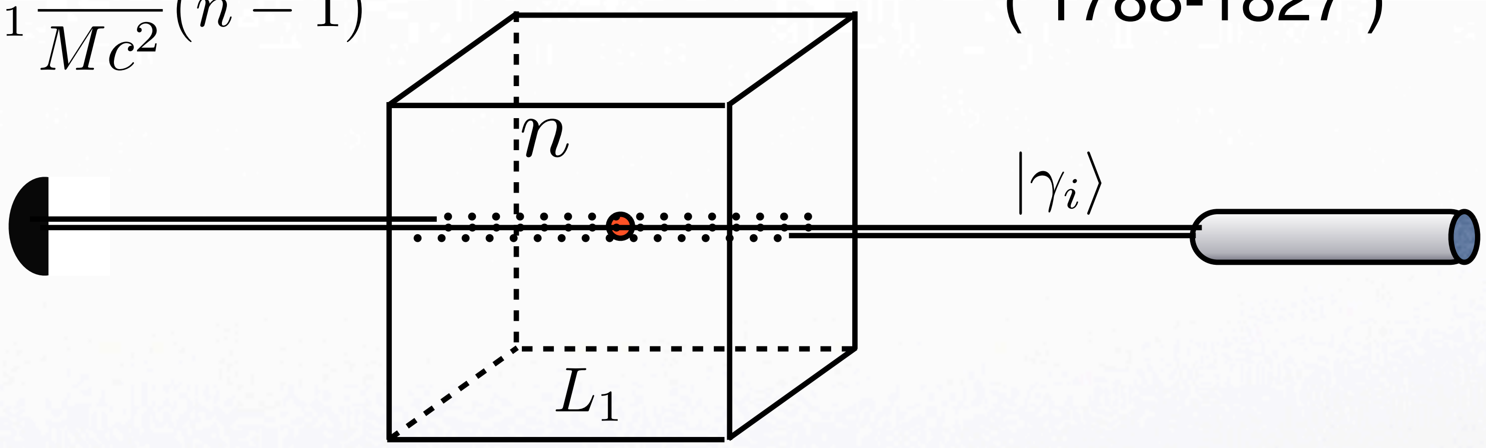
# Photon-block entanglement

$$n_2 \longleftarrow n_1$$

$$E_t = \frac{2n_1}{n_1 + n_2} E_i \qquad \text{and} \qquad E_r = \frac{n_1 - n_2}{n_1 + n_2} E_i$$

Augustine-Jean Fresnel
( 1788-1827 )

$$F_0 = \frac{4n}{(1+n)^2}\, e^{\imath n \omega L_1/c} \iff \Delta X_0 = L_1 \frac{\hbar\omega}{Mc^2}(n-1)$$




$$F_j = \frac{4n}{(1+n)^2} \frac{(n-1)^{2j}}{(1+n)^{2j}}\, e^{\imath(2j+1)n\omega L_1/c} \iff \Delta X_j = L_1 \frac{\hbar\omega}{Mc^2}(n-1+2j)$$

$$|\psi_{\leftarrow}\rangle = \sum_{j=0}^{\infty} \frac{4n}{(1+n)^2} \frac{(n-1)^{2j}}{(1+n)^{2j}}\, e^{\imath(2j+1)n\omega L_1/c}\, |\gamma_i\rangle \otimes |\Delta X_j\rangle$$





But in reality there is some back reflection. According to Fresnel's formulae, when an electromagnetic wave transits from a medium with index n_1 to one with n_2 across a plane sharp boundary, the reflected and transmitted electric fields get multiplied by .... Thus if the wave goes though the block (2 transmissions) its amplitude pick up a factor F_0 which includes the phase accrued during crossing the block. This factor is associated with the Delta X_0 translation of the block c.m. If the wave undergoes a reflection on front and back of the block j times before going on to the left, the amplitude of the transmitted E field is multiplied by F_j where you see the phase contribution from 2j crossings as well as the expected (E_r/E_i)^(2j) factor. Here is the total amplitude for the state with a left-moving photon; gamma_i is the incoming photon state. We really have an entangled state, with each different outgoing photon phase corresponding to a different block translation.

What would be the probability that the photon did not bounce internally given that it has been detected?

# Probabilities

$$p_j = \frac{16n^2}{(1+n)^4}\frac{(n-1)^{4j}}{(1+n)^{4j}} \qquad j = 0, 1, 2, \cdots \qquad p_\leftarrow \equiv \sum_{j=0}^{\infty} p_j = \frac{2n}{n^2+1}$$

$$p(j=0|\leftarrow) = \frac{p_0}{p_\leftarrow} = \frac{8n(n^2+1)}{(n+1)^4}$$

$n = 1.6$

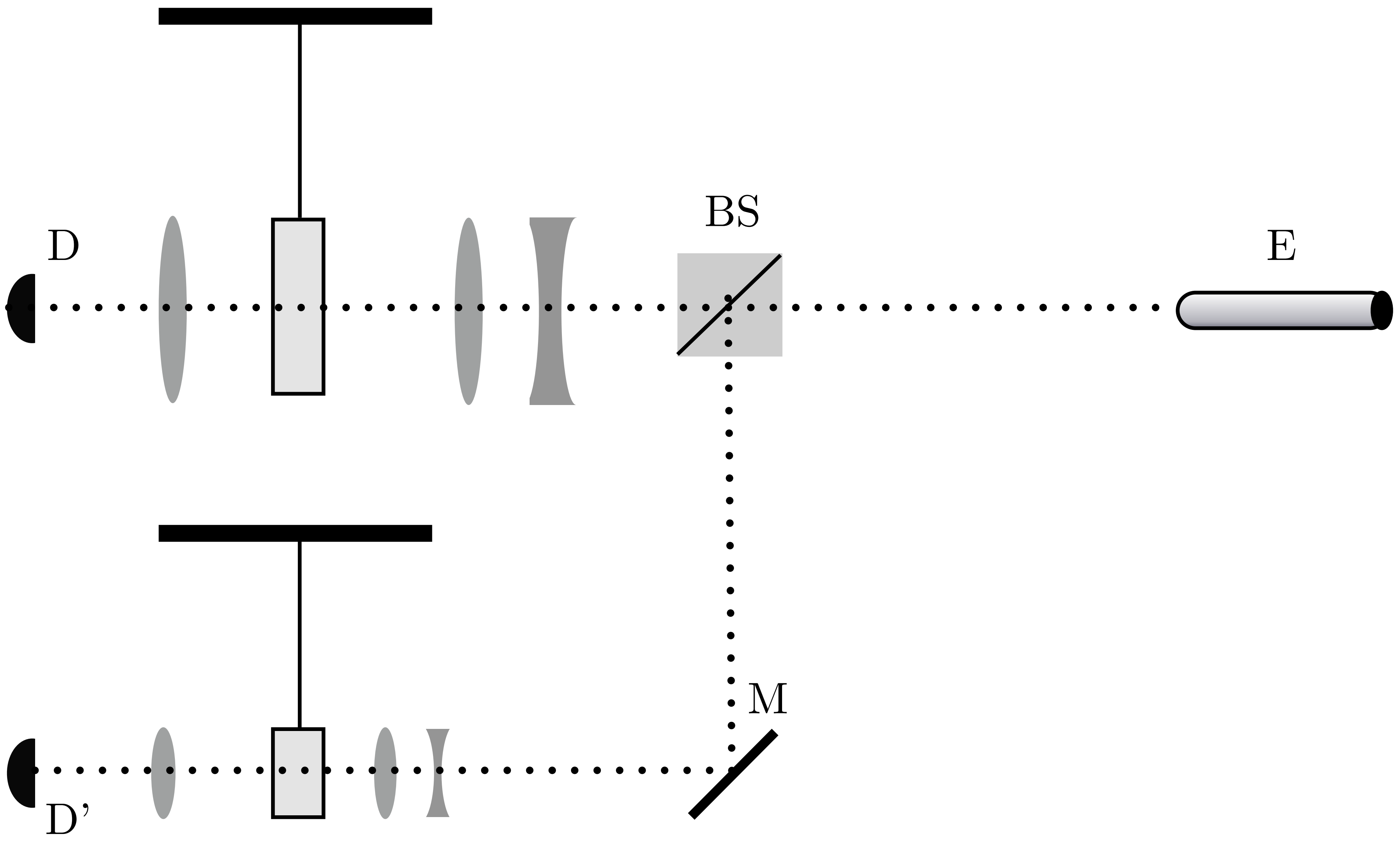
$p_0 = 0.896$ $\qquad p_1 = 0.00254$ $\qquad p_2 = 7.2 \times 10^{-6}$

$p_\leftarrow = 0.899$ $\qquad p(j=0|\leftarrow) = 0.997$





p_j is a priori probability that the photon gets through with j internal reflections. p_< is the probability the photon gets through. Period. We use n=1.6 You see the probabilities for more and more internal bounces fall off quickly. The total transit probability is 90%. The probability that no internal reflection took place when we do get a photon through is 99.7%. So if you see that a photon got through, it means the block was shifted by \delta X_0.
This last conclusion assumes that the block will translate by Delta X_0 whenever the circumstances would dictate that the photon cross it. But we argued that motion by about a Planck length is impeded. I would think, then that, with some probability, the block does not translate. But then conservation of momentum would be violated if the photon crossed the block. Something else must happen to the photon; for sake of argument let's say it is back-reflected. This would happen with some probability over and above that required by Fresnel's classical formula. Detecting the corresponding reduced transmission could reveal the quantum foam.






But the anomalous contribution may well be small.  To bring out the small amount of anomalous reflection, I propose the following arrangement.

The original system is accompanied by an analogous one based on a lighter block of like material and same thickness.  A beam-splitter and mirror assembly gives the photon a choice of two paths.  Over the upper path the back-reflection will have the anomalous part; over the lower one this last will be suppressed.  Hence were the two arms balanced and perfectly symmetric, we would expect D’ to detect the photon more often than D.  To suppress background D and D’ should be triggered after the single photon leaves E, and to give the trigger signal time to reach them before the photon a delay should be interposed before the beam-splitter.

# Sources of noise

- confusion from background light
- hits by cosmic rays
- hits by solar neutrinos
- hits by dark matter particles
- dispersion in the dielectric
- restoring force from the fiber
- Newtonian attraction between the blocks
- noise from blackbody photons
- thermal agitation of the c.m. due to molecular hits on block





All that was for an ideal experiment. In reality there is noise. Here is a partial list of possible sources. Analysis shows that with the proposed parameters all but the last two have little weight. But thermal noise is a problem.

# Noise from blackbody photons

$$N = 0.244 \left(\frac{k_B T}{\hbar c}\right)^3 = 20.3\, {T_K}^3\, \mathrm{cm}^{-3}$$

$$\lambda_{\mathrm{peak}} = \frac{1.60\, \hbar c}{kT} = \frac{0.367\, \mathrm{cm}}{T_K} \qquad p_{\mathrm{peak}} = 3.92\, k_{\mathrm{B}} T / c$$

$$\Delta p \sim T^{5/2}$$

At 4K the thermal photon momentum deposited during optical photon traversal is only 1% of that brought by the optical photon.

However, after traversal the block is left in a new state of motion with respect to that at optical photon ingress. Thus it is preferable to go to 0.5K. Wavelengths are then larger than L and the crossection is Rayleigh's: $\sigma \sim \lambda^{-4} \sim T^4$. So at 0.5K the probability of a thermal photon hit during optical photon traversal is very small.



21

Given the thermal photon density and momentum at peak of spectrum it is easy to see that at room temperature the thermal momentum deposited by photon scattering during optical photon traversal is larger than that deposited by the optical photon - experiment would fail. Note that the momentum deposited scales at T^5/2 and goes down fast at T decreases. At 4K this thermal momentum is only 1% of the optical photon momentum. There is one problem, though: The thermal momentum puts the block in a different state of motion from its original one. Makes it hard to exhibit a translation specifically due to spacetime fluctuations. If we go to 0.5K the peak wavelength is already larger than the block and geometric thermal photon cross-section will be replaced by Rayleigh's, scaling as T^4. Very rapidly the chance for a thermal photon scatter during optical photon traversal is totally negligible.

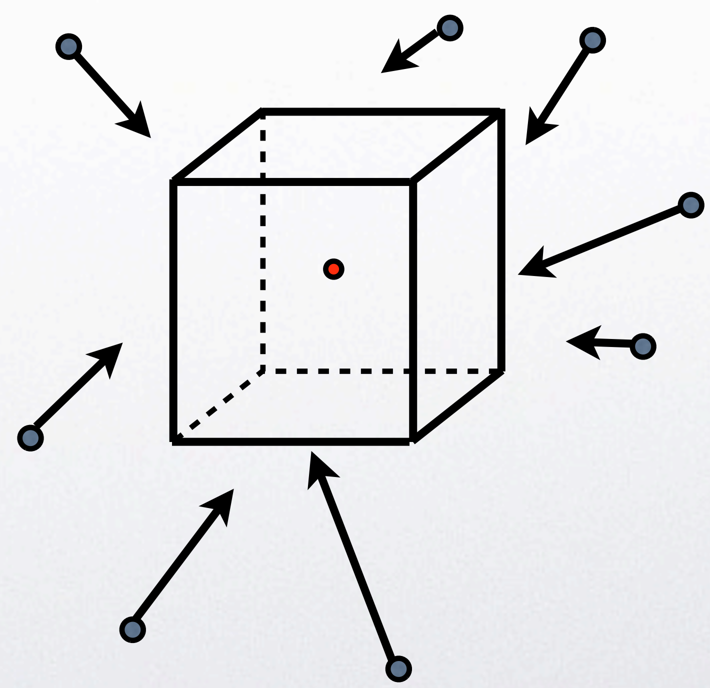
# Agitation due to gas molecules

compare $V_t = \sqrt{\dfrac{3k_{\mathrm{B}}T}{M}}$ with $\dfrac{\Delta p}{M} = \dfrac{\hbar\omega}{Mc}\left(1 - \dfrac{1}{n}\right)$

$$k_{\mathrm{B}}T_c = \frac{(n-1)^2}{3n^2}\frac{(\hbar\omega)^2}{Mc^2}$$

$$T_c \approx 10^{-29}\,^{\mathrm{o}}\mathrm{K}$$

but the game is not up!





22

V_t is the root mean square speed of the block c.m. in equilibrium at temperature T. Compare this with the speed given the c.m. by the transiting photon. They will be equal at the critical temperature T_c. With the numbers quoted earlier for \omega, M and n we get T_c=10^{-29} K, an impossible goal. But actually the situation is not as bad as it seems. The aforementioned thermal jitter of the c.m. is maintained by the collisions of ambient gas molecules with the block. The speed V_t which we estimated above is the random c.m. speed acquired after so many collisions that thermodynamic equilibrium has been reached between gas and block. However, between collisions the block's c.m. moves uniformly, and can be said to be at rest in some reference frame. In a sufficiently good vacuum, collisions are very rare and the c.m. velocity acquired by the block from one photon will not get changed much during its traversal. It then becomes irrelevant that there is a large equilibrium thermal noise of the c.m. The obstruction to the experiment is not the c.m. thermal motion per se, but the individual changes to it during the photon transit.

# Beating the thermal jitter

$$L^2 = 2(2L_1 L_2 + {L_2}^2)$$

$$\Pi \equiv \frac{\varrho}{\mu}\sqrt{\frac{3k_\mathrm{B}T}{\mu}}\, L^2\, \frac{nL_1}{c} \ll 1$$

$$P \ll \underbrace{\frac{1}{nL^2L_1}\sqrt{\frac{kT_\mathrm{B}\mu c^2}{3}}}_{3.3\times 10^{-10}\,\mathrm{Pa}}$$

$\mu = 4$ a.m.u.

block density $= 6\,\mathrm{g\,cm^{-3}}$

$L_1 = 1\,\mathrm{mm}$

$L_2 = 5\,\mathrm{mm}$

$\lambda = 445\,\mathrm{nm}$

$n = 1.6$

$T = 0.5\,^\circ\mathrm{K}$



How frequent are these? Define L^2 as the surface area of the block. Look at this product. Here we have the thermal speed of a typical air molecule. \mu is the molecular mass. Then the number density of air molecules times their typical speed times L^2 is the flux onto the block. We multiply the flux by the transit time of the photon. Require this average number of hits to be small compared to 1. Thus Pi is really the probability of a hit during transit. With the ideal gas law we replace rho by P getting this criterion. Helium would be an environment gas of choice. So \u=4 a.m.u. We further take these values. We thus need to have the pressure well below 8 x 10^{-9} Pa. It turns out one has to work at low temperatures 0.5 K. Then the requirement is pressure below 3.3 x 10^{-10} Pa. In fact today, using off the shelf equipment and some care, it is possible to reach 10^{-11} Pa. This means only one out of 30 transits is accompanied by a hit. The low temperature is required to control thermal photon noise - low pressure won't help there.

**Bottom line: Experiment can be done.**