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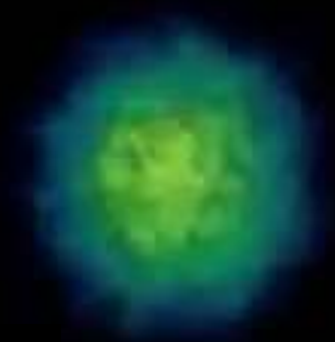
Magnetic Monopoles
Theory and Experiment



Arttu Rajantie
HEP Seminar, University of Liverpool
6 November 2013

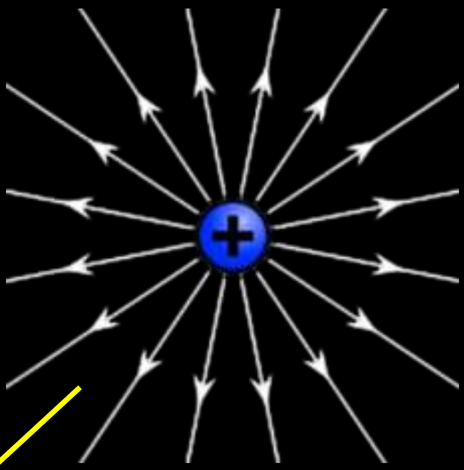
Maxwell Equations

$$\vec{\nabla}\cdot\vec{E} = \rho_{\rm E}$$

$$\vec{\nabla}\cdot\vec{B} = \rho_{\rm M}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} - \vec{j}_{\rm M}$$

$$\vec{\nabla}\times\vec{B} = \frac{\partial\vec{E}}{\partial t} + \vec{j}_{\rm E}$$



Duality $\vec{E} \leftrightarrow \vec{B}$

Electromagnetic Potentials

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}, \vec{B} = \vec{\nabla} \times \vec{A}$$

- Quantum Mechanics:

  Complex phase of the wave function couples to $\vec{A}$

$$i\hbar \frac{\partial}{\partial t}\psi = -\frac{\left(\hbar\vec{\nabla} + ie\vec{A}\right)^2}{2m}\psi + e\phi\psi$$

- Sourceless magnetic field: No monopoles?

$$\vec{\nabla} \cdot \vec{B} = \vec{\nabla} \cdot \left(\vec{\nabla} \times \vec{A}\right) = 0$$

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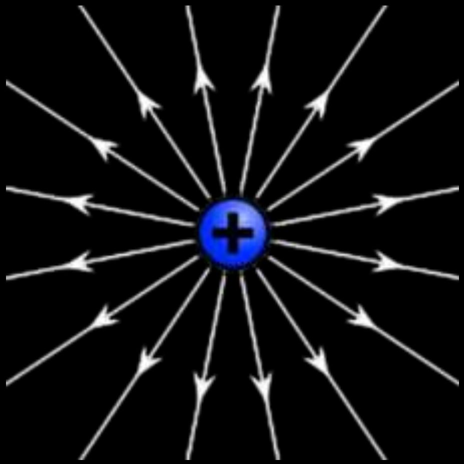
Dirac Monopole (1931)

- Vector potential $\vec{A}(\vec{r}) = \frac{g}{4\pi|\vec{r}|}\frac{\vec{r}\times\vec{n}}{|\vec{r}| - \vec{r}\cdot\vec{n}}$
- Singularity along $\vec{n}$ (Dirac string):
  - Carries magnetic flux $\Phi = g$
  - Classically observable: Induces current in wire loop

Dirac Monopole (1931)

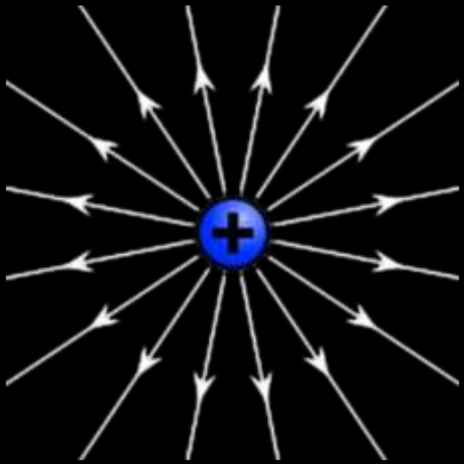
- Vector potential $\vec{A}(\vec{r}) = \frac{g}{4\pi|\vec{r}|}\frac{\vec{r}\times\vec{n}}{|\vec{r}| - \vec{r}\cdot\vec{n}}$
- Singularity along $\vec{n}$ (Dirac string):
  - QM: Complex phase defined modulo 2π
  - String unobservable if $g = g_0 = 2\pi/e$

Dirac Monopole (1931)



- If all electric charges satisfy the Dirac quantisation condition $e \in \frac{2\pi}{g}\mathbb{Z}$, string not detectable: Pointlike monopole
- Explanation of charge quantisation?
- Conversely g must be integer multiple of $g_0 = \frac{2\pi}{e_0}$

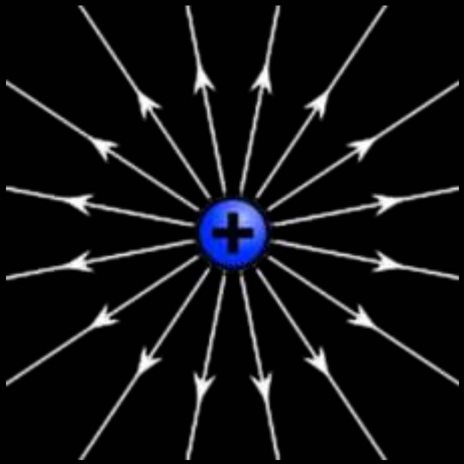
Dyons



- Particles with electric charge e and magnetic charge g
- Any two particles have to satisfy the quantisation condition (Schwinger 1966):

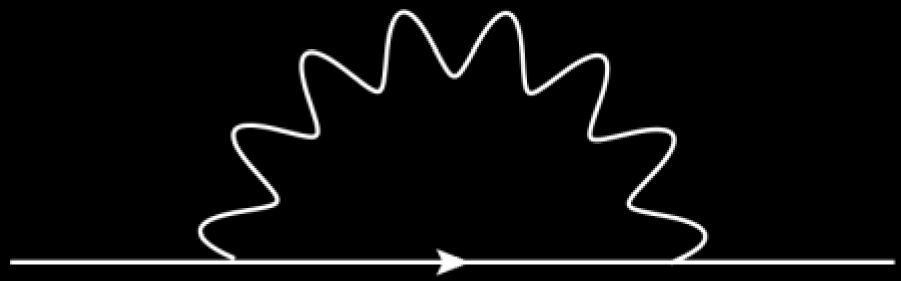
$$e_1 g_2 - e_2 g_1 \in 2\pi\mathbb{Z}$$

Mass Estimate



- Magnetic Coulomb field: $\vec{B}(\vec{r}) = \frac{g}{4\pi}\frac{\vec{r}}{|\vec{r}|^3}$
- Magnetic charge localised at a point
- Divergent energy: $E = \int d^3x \frac{\vec{B}^2}{2} \sim g^2\Lambda \sim \frac{\Lambda}{e^2}$

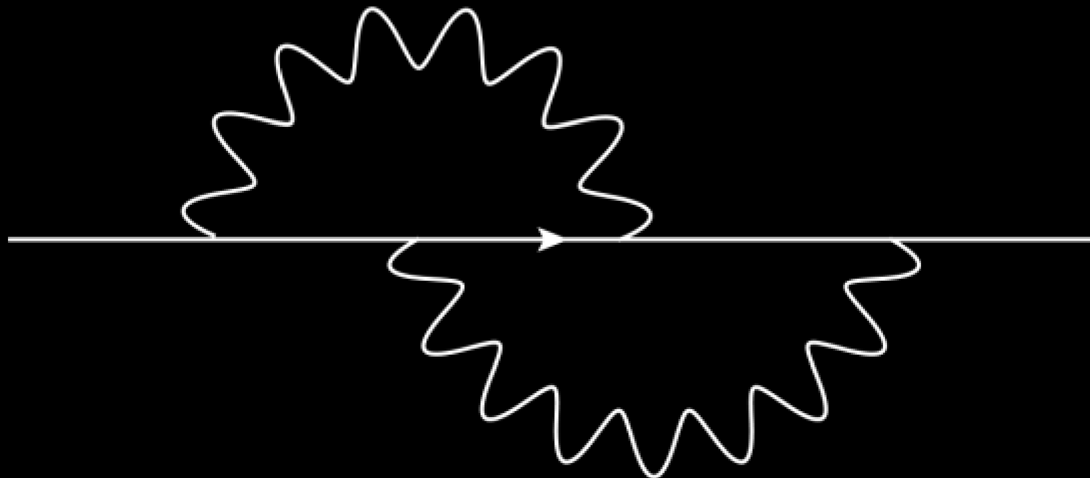
QFT of Monopoles



- Full quantum field theory calculation: Monopole loops
- Compare with electron mass correction

$$\delta m = -\frac{e^2}{2\pi^2} m \log \frac{\Lambda}{m} \ll e^2 \Lambda$$

QFT of Monopoles



$\propto g^4$

- Difficult to formulate:
  Two vector potentials (Schwinger 1975)
- Strong coupling $g = \frac{2\pi}{e} \gg 1$
  Non-perturbative!

‘t Hooft-Polyakov Monopole (1974)

- Georgi-Glashow model: SU(2)+adjoint Higgs

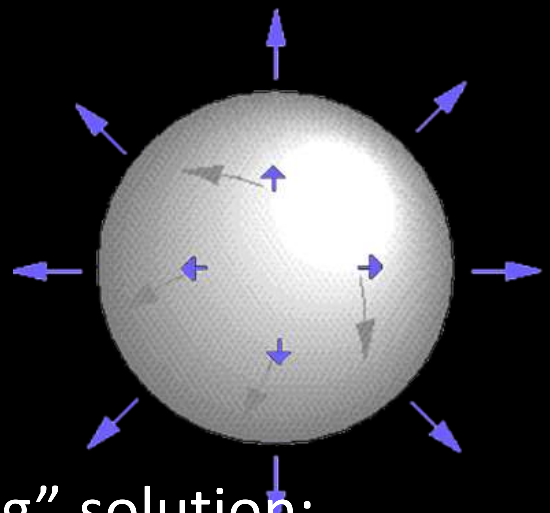
$$\begin{aligned}\mathcal{L} &= -\mathrm{Tr}F^{\mu\nu}F_{\mu\nu} + \mathrm{Tr}[D_\mu,\Phi][D^\mu,\Phi] \\ &\quad -m^2\mathrm{Tr}\Phi^2 - \lambda\mathrm{Tr}\Phi^4\end{aligned}$$

- $\Phi \neq 0 \Rightarrow$ Symmetry breaking $SU(2) \rightarrow U(1)$
- Electrodynamics with magnetic field

$$B_i = \frac{1}{2}\epsilon_{ijk}\mathrm{Tr}\hat{\Phi}\left(F_{jk} - \frac{i}{2e}[D_j,\hat{\Phi}][D_k,\hat{\Phi}]\right)$$

- Sourceless ($\vec{\nabla}\cdot\vec{B} = 0$) except when $\Phi = 0$

't Hooft-Polyakov Monopole (1974)

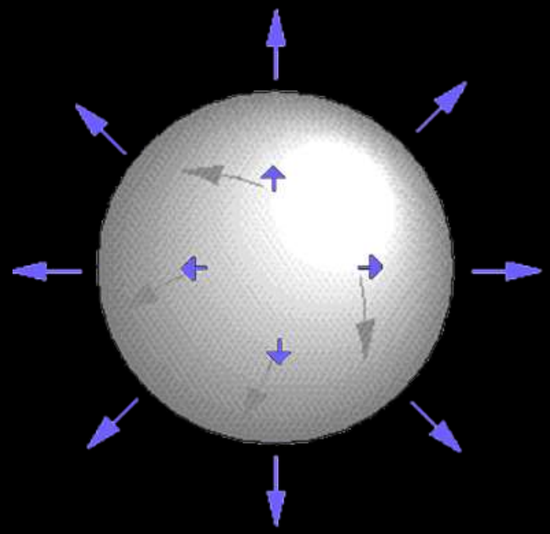


- Smooth "hedgehog" solution:

$$\Phi^a \propto x_a, \quad A_i^a \propto \epsilon_{iaj} x_j$$

- Magnetic charge $g = \int d\vec{S} \cdot \vec{B} = 2g_0 = 4\pi/e$
- Finite mass $M \approx \frac{4\pi v}{e} \sim \frac{m}{e^2}$

‘t Hooft-Polyakov Monopole (1974)

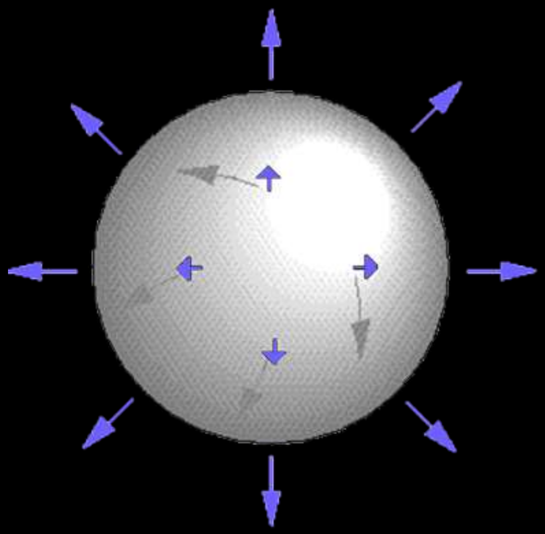


- Montonen-Olive duality conjecture (1977):

Electric ↔ Magnetic

$$e \leftrightarrow 1/e$$

't Hooft-Polyakov Monopole (1974)

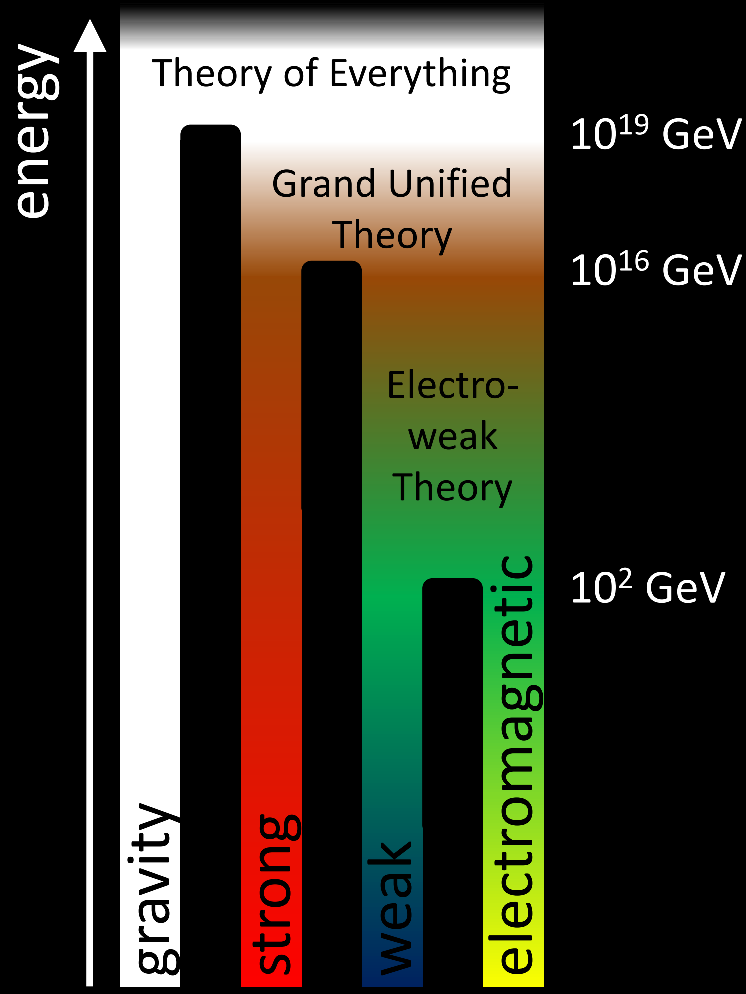


- The same solution exists whenever simple Lie group broken to something with a U(1) factor:

Grand Unification

Grand Unification

- Standard Model:
  EM & weak forces unified above 100 GeV
- Grand Unified Theory (GUT):
  Electroweak & strong forces unified above 10^{16} GeV
  - e.g.
  $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$

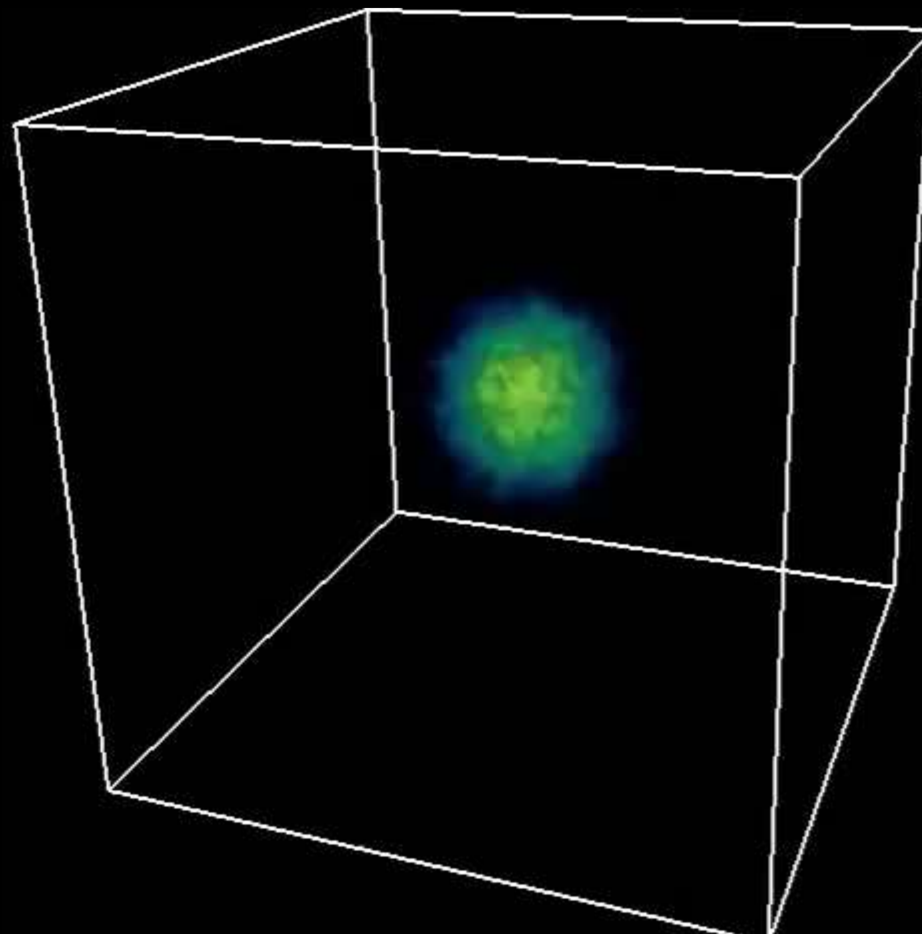


GUT Monopoles

- Generic prediction of GUTs (and ToEs)
- Mass typically at GUT scale $M \sim 10^{17}$ GeV
- Lightest singly-charged

$$g = g_0 = \frac{2\pi}{e}$$

- Catalyse proton decay (Rubakov, Callan 1981)

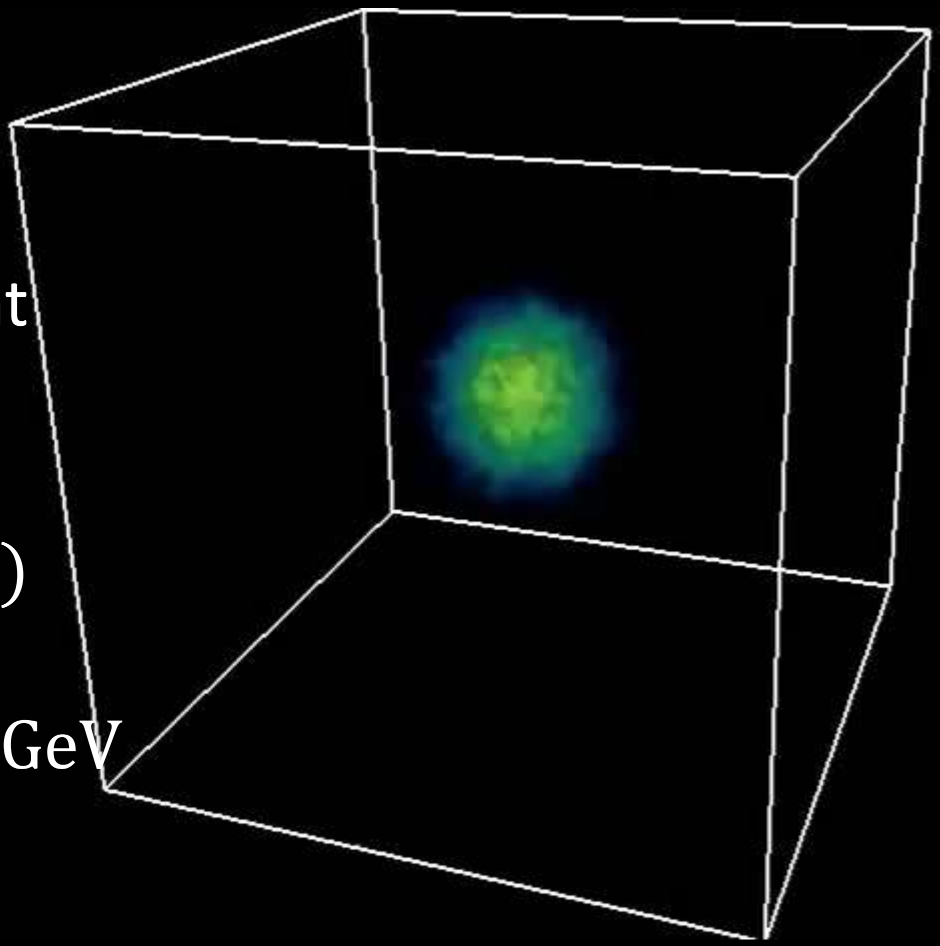


Simulation of a quantum monopole (AR 2005)

Visualisation: www.vapor.ucar.edu

GUT Monopoles

- More complex GUTs, e.g. $SO(10)$
- Monopoles with different charges
- Can be lighter: $SU(4) \times SU(3) \times SU(3)$ has multiply-charged monopoles with $M \sim 10^7 \mathrm{GeV}$ (Kephart et al)



Simulation of a quantum monopole (AR 2005)

Visualisation: www.vapor.ucar.edu

String Theory Monopoles

- S-duality:
  Any superstring theory has magnetic monopoles

- Kaluza-Klein monopole (Gross&Perry, Sorkin):
  Compactified dimension $\leftrightarrow$ U(1) of electrodynamics

- Typical mass $M \sim \frac{M_{\mathrm{Pl}}}{e} \sim 10^{20}\ \mathrm{GeV}$

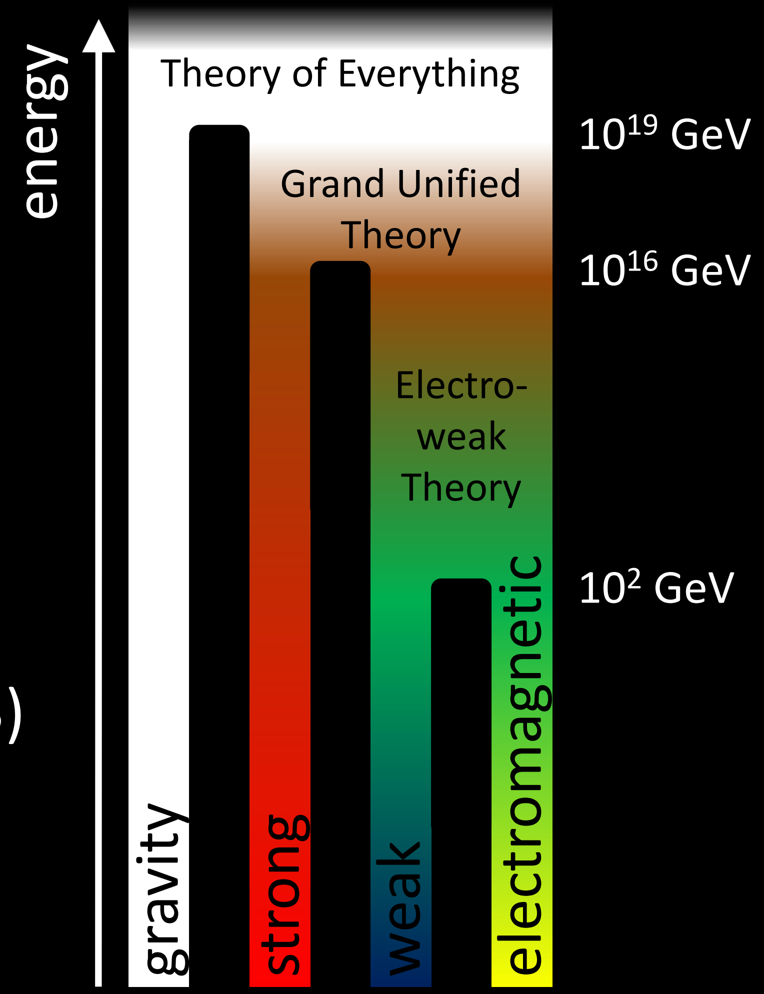
Cho-Maison Monopole (1996)

- Dirac solution generalised to electroweak theory
- Needs a source particle
- Demonstrates that TeV-scale monopoles are possible
- Twice the Dirac charge

$$g = 2g_0 = \frac{4\pi}{e}$$

- Mass estimate (Cho&Pinfold 2013)

$$M \gtrsim \frac{m_W}{e^2} \sim \text{few TeV}$$

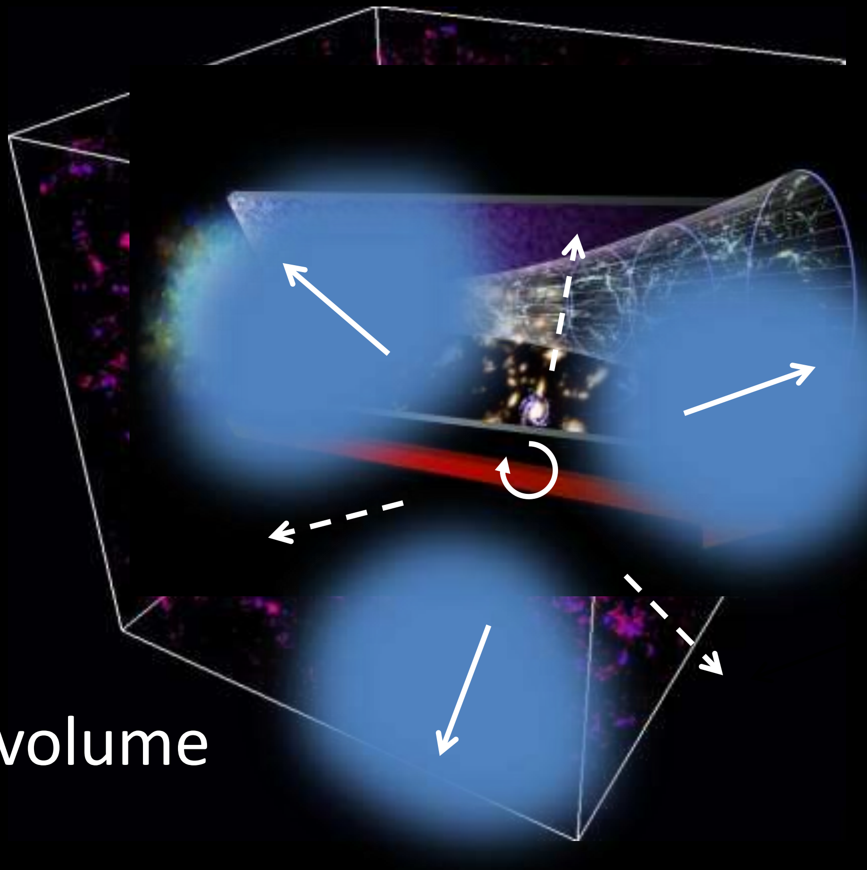


Probes of New Physics

- Strong, precisely known EM interactions
- Electrodynamics $\Rightarrow$ Stable particles
- Interaction with charged fermions depends on core structure (Rubakov, Callan 1981)
- Properties related to beyond the SM physics:
  - Unification of forces
  - Fundamental properties of electromagnetism

Cosmic Monopoles

- Hot Big Bang:
  GUT symmetry breaks
  in a phase transition
- The Higgs field chooses
  a direction randomly
- Kibble (1976):
  Monopoles form,
  at least one per horizon volume
  $\rightarrow n_{\mathrm{mon}} \sim H^{-3}$



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Cosmic Monopoles

- Monopoles annihilate until they cannot find partners: Density decreases to

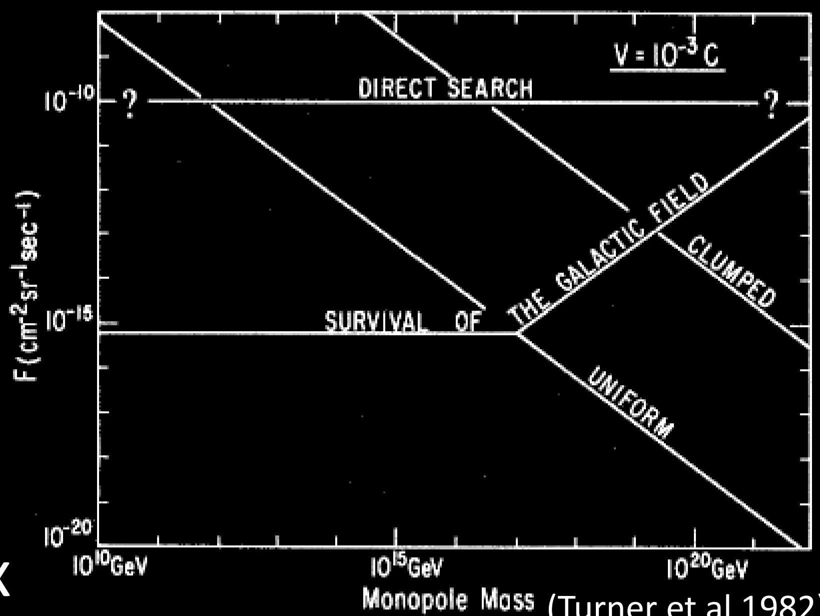
$$n_{\text{mon}} \sim 10^{-8} \left(\frac{M}{10^{17}\ \text{GeV}} \right) T^3 \sim 10^{-1} \left(\frac{M}{10^{17}\ \text{GeV}} \right) \text{m}^{-3}$$

(Zel'dovich & Khlopov 1979, Preskill 1979)

- Mass density higher than observations unless $M \lesssim 10^{10}\text{GeV}$: Monopole problem
- Guth (1981): Inflation wiped monopoles away
  - Monopole production after inflation?

Parker Bound (1970)

- Galactic magnetic fields $B \sim 3\mu G$
- If $M \lesssim 10^{17}\mathrm{GeV}$, this creates a magnetic current, which dissipates the field
- Sets an upper bound on flux



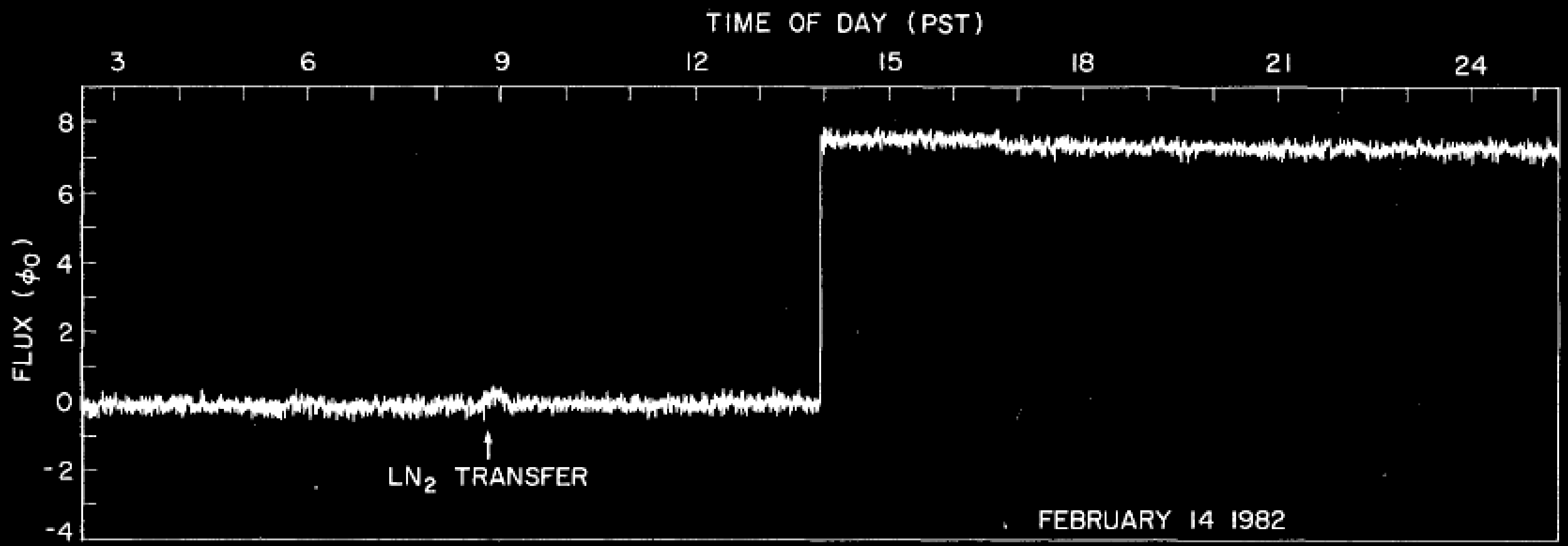
(Turner et al 1982)

$$F = \frac{nv}{4\pi} \lesssim 10^{-15}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}\mathrm{sr}^{-1}$$

- Extended Parker bound (Adams et al 1993)

$$F \lesssim 10^{-16}\left(\frac{M}{10^{17}\ \mathrm{GeV}}\right)\mathrm{cm}^{-2}\ \mathrm{s}^{-1}\mathrm{sr}^{-1}$$

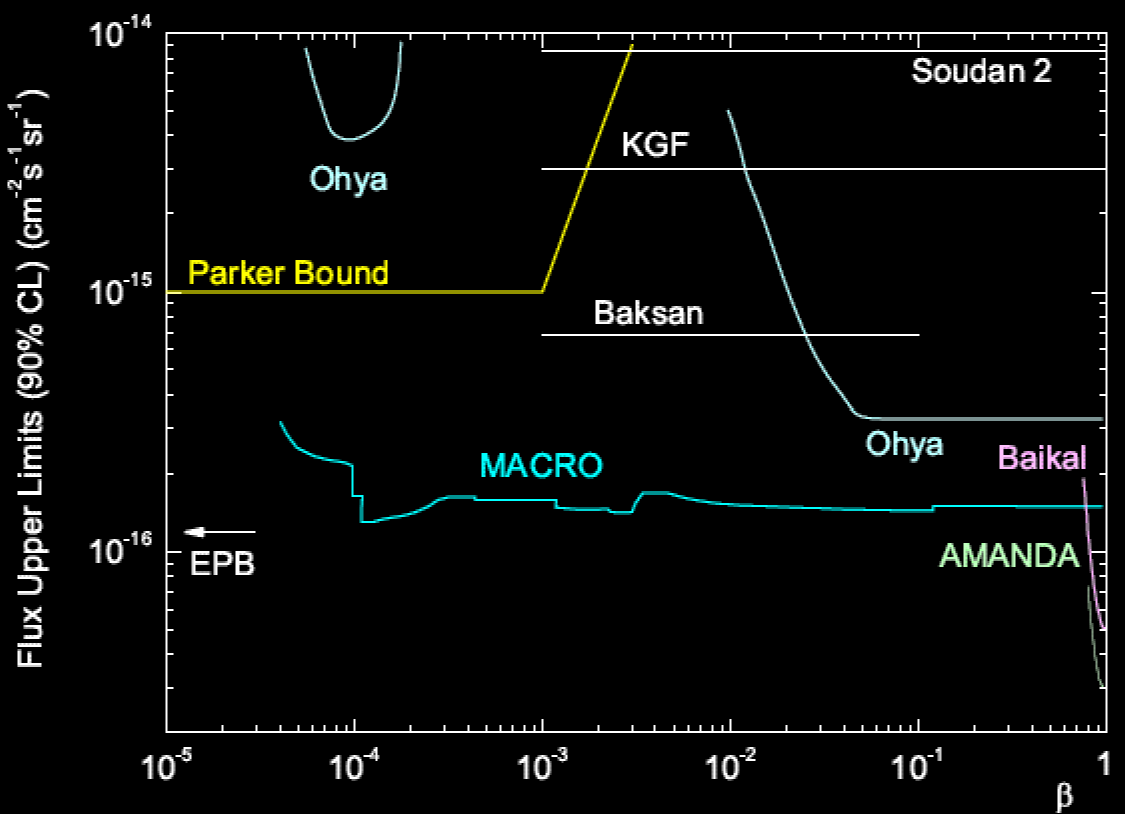
Cosmic Rays



(Cabrera 1982)

- Early detections:
  - Berkeley 1975, Stanford 1982, Imperial 1986
  - All turned out to be false

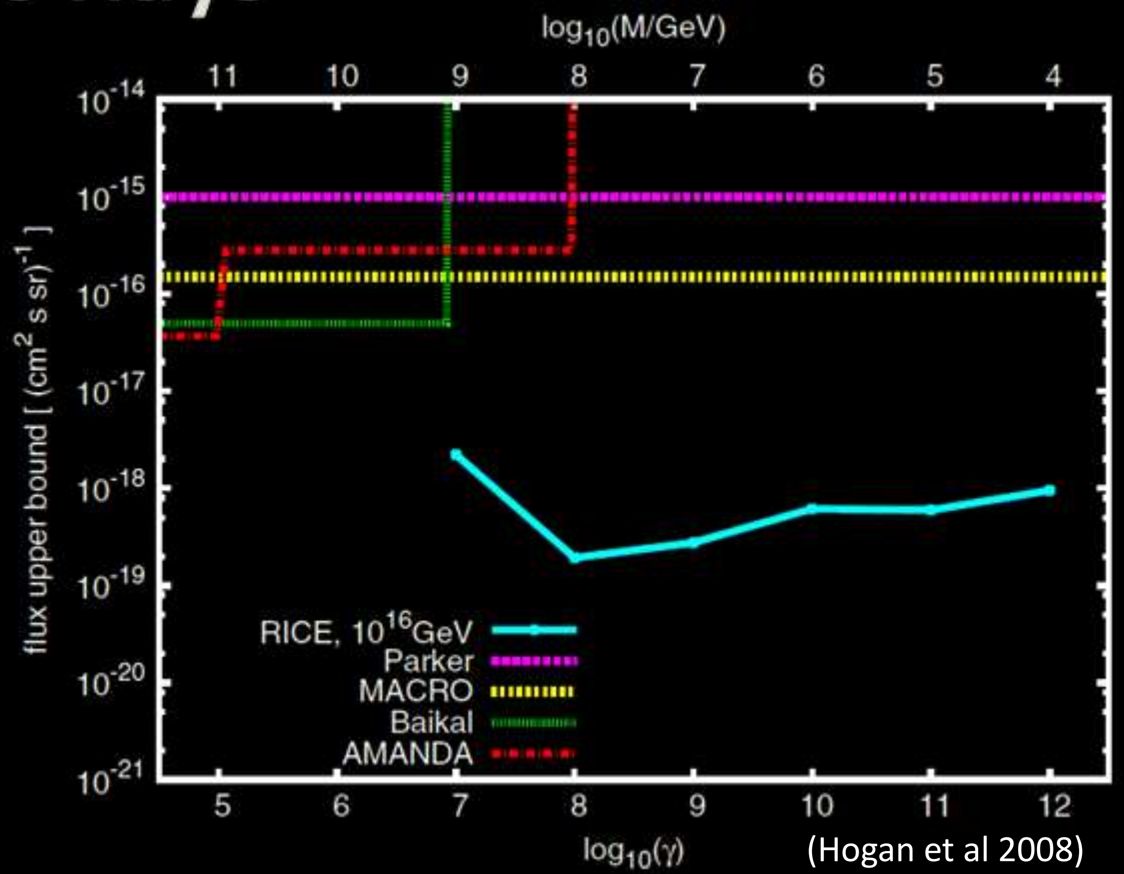
Cosmic Rays



(Giacomelli et al 2011)

- MACRO (Gran Sasso, Italy, 1989 – 2000):
  - Upper bound $F \lesssim 10^{-16}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}\mathrm{sr}^{-1}$ over wide mass range

Cosmic Rays



(Hogan et al 2008)

- RICE (South Pole):
  - Intermediate mass monopoles $F \lesssim 10^{-18}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}\mathrm{sr}^{-1}$

Monopoles Trapped in Matter

- Moon rocks (Alvarez et al 1970)
- Deeply buried rocks (Kovalik & Kirschvink 1986)
- Meteorites (Jeon & Longo 1995)
- Polar volcanic rocks (Bendtz et al 2013)

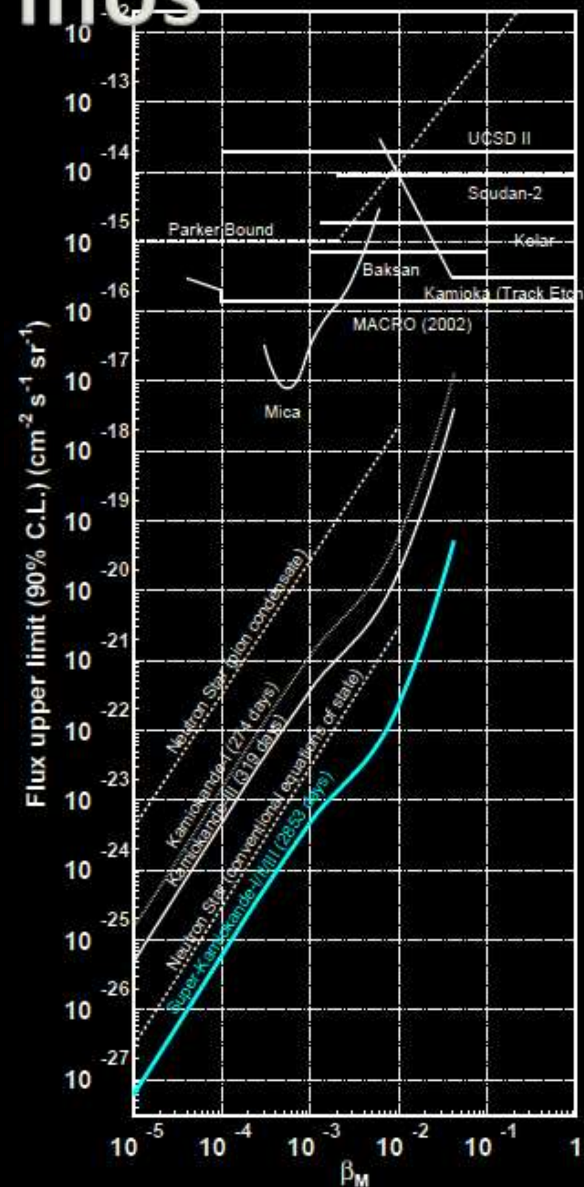
- Bound: Fewer than $\sim 10^{-28}$ per nucleon
- Hard to draw strong conclusions

Monopole-Induced Neutrinos

- Monopoles accumulate in the Sun
- GUT monopoles catalyse p decay:
  Produces neutrinos (Rubakov 1981)
  Super-K bound (2012):

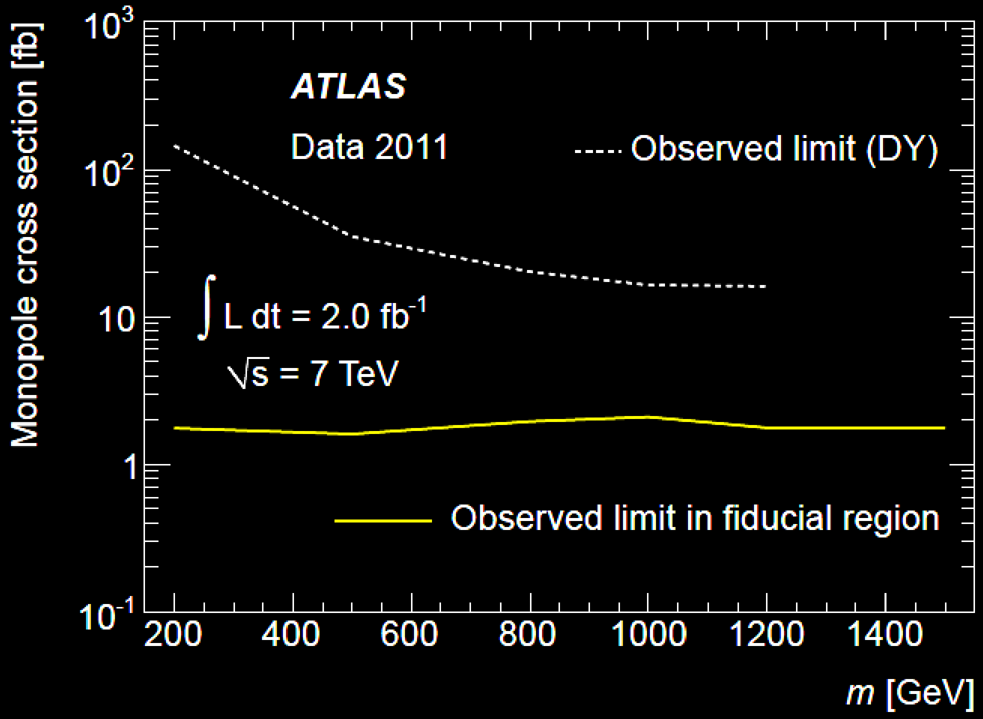
$$F \lesssim 10^{-23} \left(\frac{\beta_M}{10^{-3}}\right)^2 \mathrm{cm}^{-2}\,\mathrm{s}^{-1}\mathrm{sr}^{-1}$$

- Model-dependent



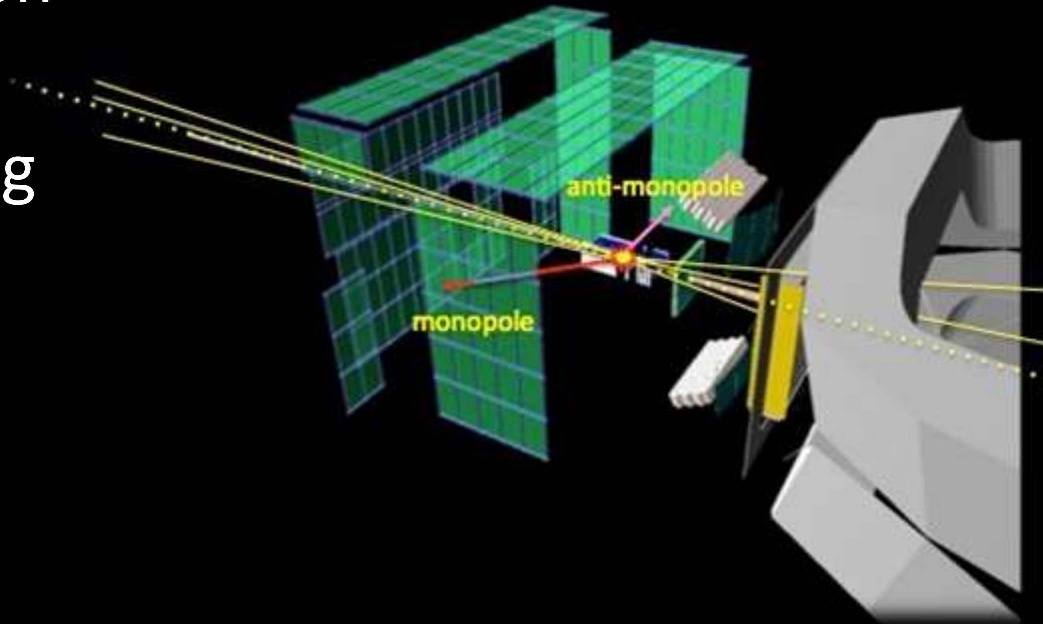
Accelerator Searches

- Direct searches:
  - OPAL (LEP) 2008: $\sigma < 50$ fb for $M \lesssim 100$ GeV
  - CDF (Tevatron) 2006: $\sigma < 200$ fb for $M \lesssim 700$ GeV
  - ATLAS (LHC) 2012: $\sigma < 16 - 145$ fb for $M \lesssim 1$ TeV



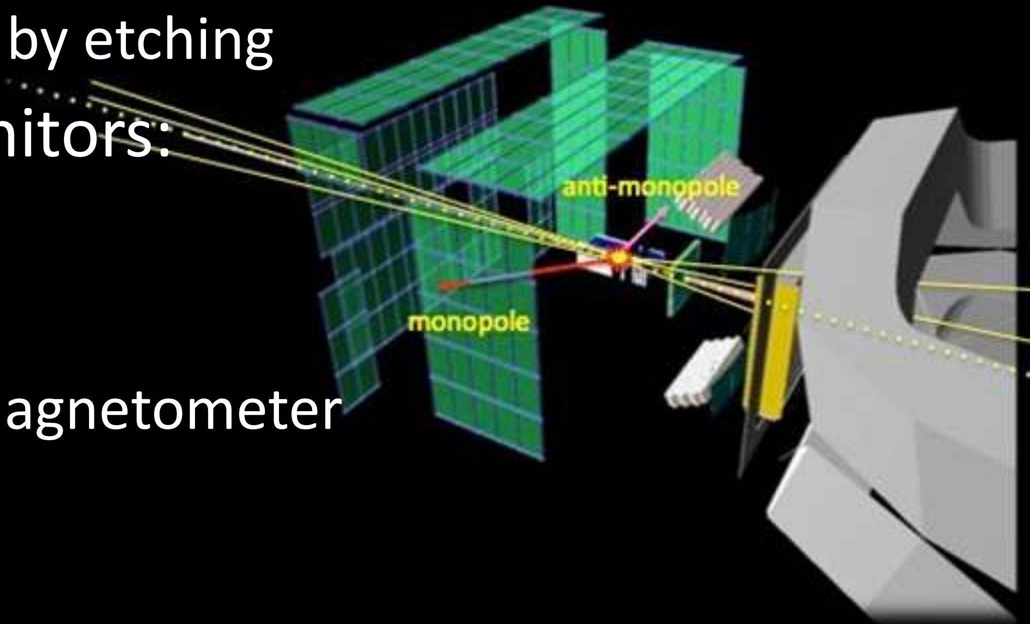
MoEDAL

- Monopole and Exotics Detector at the LHC
- 42 physicists from 11 countries
- Shares LHC intersection point 8 with LHCb
- Will be installed during the long shutdown

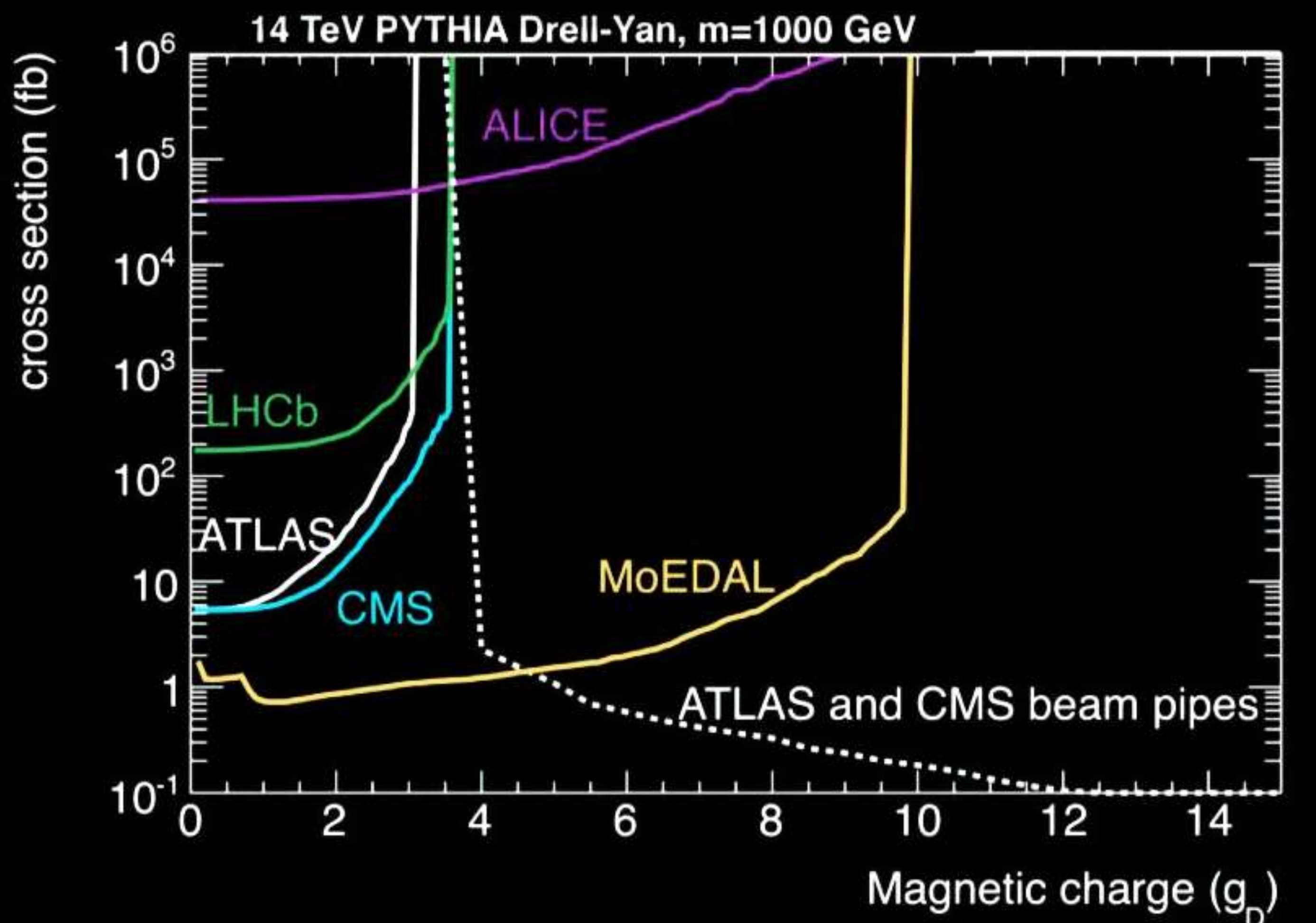


MoEDAL Detectors

- Nuclear track detectors:
  - 10 layers
  - Total area $\sim 10 \times 25\ \mathrm{m}^2$
  - Particle tracks revealed by etching
- TimePix radiation monitors:
  - Real time
- Trapping detectors:
  - Analysed with SQUID magnetometer



MoEDAL


14 TeV PYTHIA Drell-Yan, m=1000 GeV
cross section (fb)
Magnetic charge (g_D)
ALICE
LHCb
ATLAS
CMS
MoEDAL
ATLAS and CMS beam pipes
10^6
10^5
10^4
10^3
10^2
10
1
10^-1
0
2
4
6
8
10
12
14

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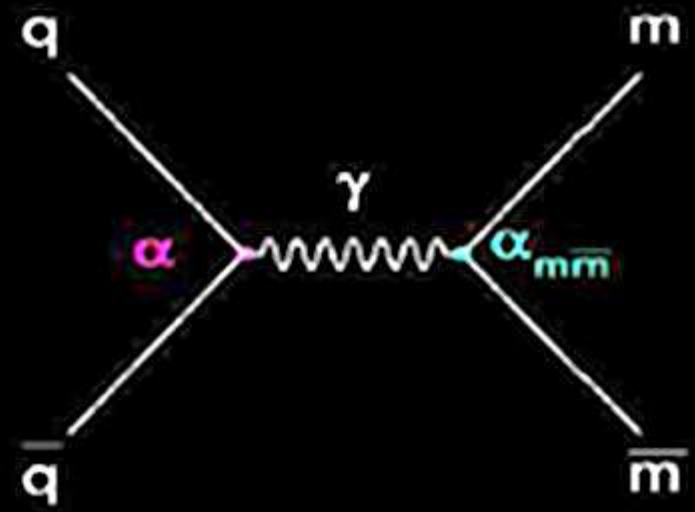
Production Cross Section

- Accelerator experiments can only constrain σ: Can we predict it from theory?
- Drell-Yan process:



- Large theoretical uncertainties: Perturbation theory not applicable

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Exponential Suppression?

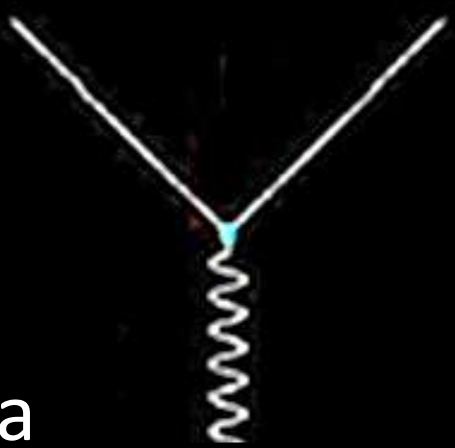
- Semiclassical estimate (Drukier&Nussinov 1982):
  - Production cross section $\sim e^{-2/\alpha}$?
  - Based on entropy arguments / no of quanta
- Numerical calculation (Demidov&Levkov 2011):
  - Semiclassical Rubakov-Son-Tinyakov method with complex field configurations
  - Confirms exponential suppression for kinks in 1+1D at weak coupling

Beyond Semiclassical

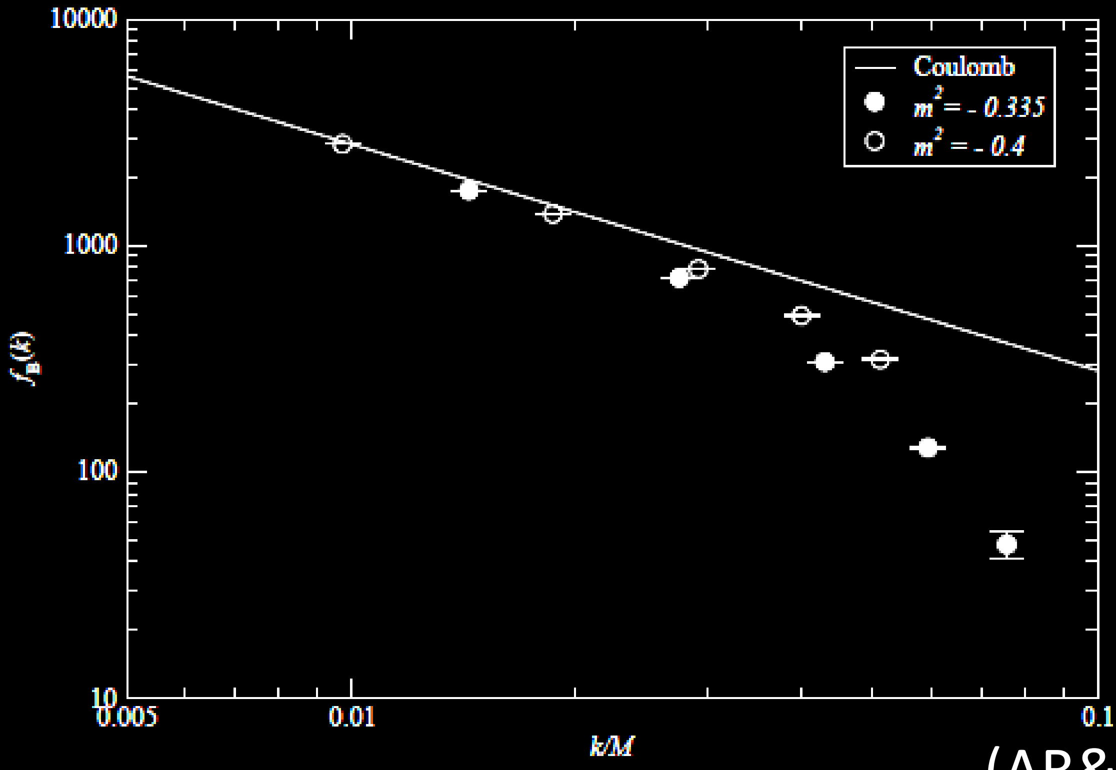
- Lattice calculation of monopole form factor (AR&Weir 2012)

$$\vec{f}(\vec{p}_1, \vec{p}_2) = \langle \vec{p}_2 | \vec{B} | \vec{p}_1 \rangle$$

- ~ monopole-photon vertex
- Crossing symmetry:
  Related to production amplitude
  by analytic continuation
- Always tricky with numerical data



Results for weakish coupling $(\lambda = 0.1)$



(AR&Weir 2012)

- Suppression at high k – production rate?

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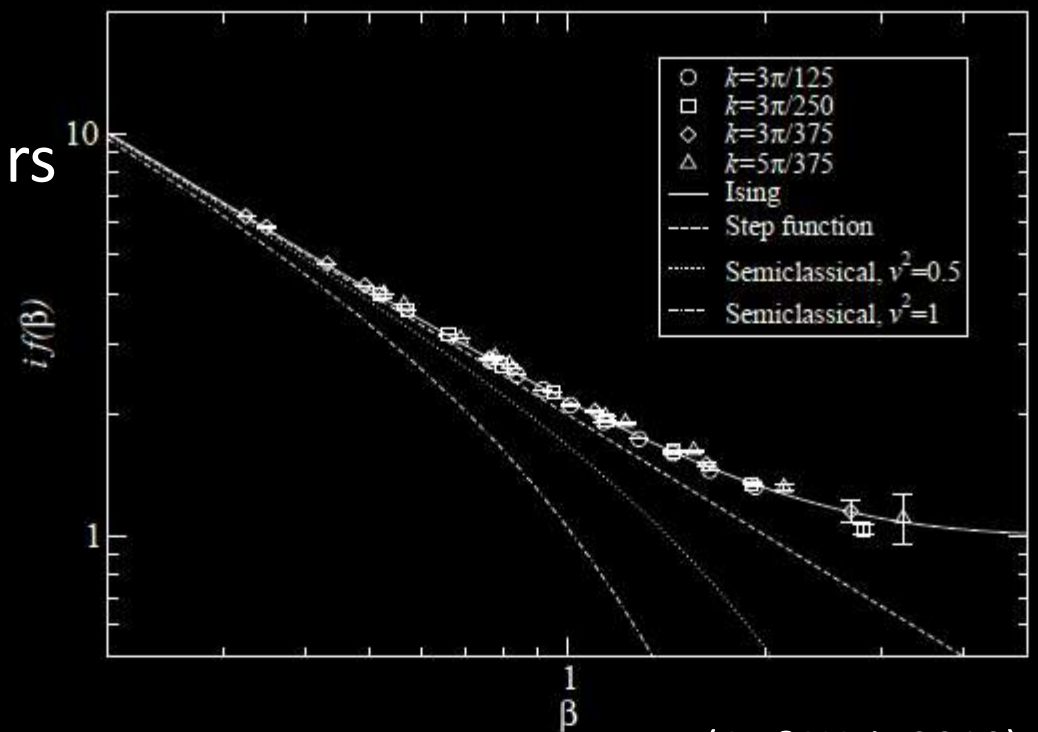
Stronger Coupling?

- 1+1D Kinks:
  Suppression disappears at strong coupling



(AR&Weir 2010)

- Monopoles:
  - Two couplings e, λ
  - e fixed
  - Numerical calculation needed
- Singular Cho-Maison monopoles?

Summary

- Monopoles are among the best motivated new particles
  - Would open up a window to exciting new physics
- Cosmic monopoles: Stringent bounds
- TeV-scale Cho-Maison monopoles possible
- Theoretical understanding still poor
  - Exponential suppression of production rate?
  - Perturbation theory not applicable
  - Numerical simulations – analytic continuation?
- Introductory review: Contemp. Phys. 53 (2012) 195